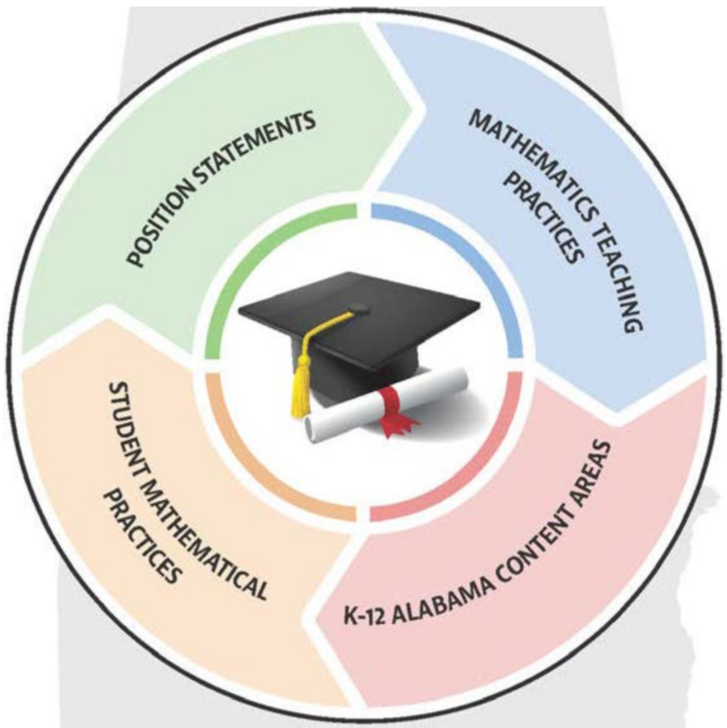
MATHEMATICS CURRICULUM FRAMEWORK



MATHEMATICS
CURRICULUM FRAMEWORK

Developed 2020-2021
Implemented 2021-2022

Mountain Brook Schools
32 Vine Street
Mountain Brook, AL 35213

MATHEMATICS
CURRICULUM FRAMEWORK

Mountain Brook Schools
Board of Education

GENERAL INTRODUCTION

The *2020 Mountain Brook Curriculum Framework for Mathematics* defines the knowledge and skills students should know and be able to do after each course and upon graduation from high school. Mastery of the standards enables students to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics (National Council of Teacher of Mathematics [NCTM], 2018). Courses within the *2020 Mountain Brook Curriculum Framework for Mathematics* are organized into Alabama Content Areas which are adapted from those present in the draft of the NAEP *2025 Mathematics Framework*. High school courses also incorporate recommendations for the Essential Concepts as identified by the National Council of Teachers of Mathematics (2018) and other documents. All standards contained in this document are:

- aligned with college and work expectations;
- written in a clear, understandable, and consistent format;
- designed to include rigorous, focused, and critical content and application of knowledge through high-order skills;
- formulated upon strengths and lessons of current Alabama standards;
- informed by high-performing mathematics curricula in other countries to ensure all students are prepared to succeed in our global economy and society; and
- grounded on sound, evidence-based research.

Mountain Brook Schools is committed to the mastery of the standards so students can build a solid foundation of knowledge, skills, and understanding in mathematics.

2020 Mathematics Curriculum Committee

Brookwood Forest Elementary
K-Jennifer Jinnette
K-Amanda Potaczek
K-Katie Seeger
K-Tara Smith
K-Perry Wright
1-Sammye Davis
1-Stacey Kirkpatrick
1-Carrie Knight
1-Brooke Rice
1-Mary Claire Singleton
2-Caroline Peek
2-Ashley Scott
2-Tanishia Sims
2-Kristin Williams
3-Kristin Davis
3-Dawn Elsberry
3-Linda Mason
3-Kelly Stout
3- Eleanor Walker
4-Natalie Borland
4-Ann Scott Cohen
4-Laura Frenz
4-Lane Walker
5-Jolie Welner
6-Virginia Moore
Parent- Margaret Anne Schilder
AP-Ashley Crossno

Cherokee Bend Elementary
K- Caroline Holley
K-Lyndsi Kirk
K- Leah Saab
K- Hannah Umphrey
K-Bethnay White
1-Kelly Anderson
1-Robyn Gaut
1-Trisha Humphries
1-Jennifer Friday
1-Meredith Lusco
2-Cynthia Echols
2-Kelley Finley
2-Christine MacPherson
2-Taylor McLean
3-Kelbe Byars
3-Danean Davis
3-Jacques Feagins
3-Maggie Helms
4-Beth Dean
4-Anna DeBell
4-Shelley Hunt
4-Sally Till
5-Barbara Parker
6-Lane Kennedy
Parent- Byron Woods
AP-Carla Dudley

Crestline Elementary
K--Phyllis Farrar
K- Melanie Hennessey
K- Emily McGuire
K- Johnna Noles
K- Michelle Ramsey
K-Kristin Seitz
K-Olivia Spurlock
1-Rachel Anderson
1-Greer Black
1- Marlyss Green
1-Sarah Black
1- Deborah Holder
1- Chelsey Summerrow
1-Lindsay Trucks
2-Tracey Barringer
2-Erin Cain
2-Allison Davis
2-Kay Haley
2-Beth McKinley
2-Christy Neely
2-Becca Pigg
3- Kelsey Long
3- Kelsey Manley
3- Kelly Mitchell
3- Carly Morgan
3-Laura Rives
3-Kate Snow

2020 Mathematics Curriculum Committee Cont'd

Crestline Elementary Cont'd
4-Sally Baker
4-Melissa Crawl
4-Caroline Ferrarone
4-Scott McKerley
4-Bradley O'Neill
4-Jennifer Preston
5-Kendra Bierbrauer
5-Cindy Carlisle
6-Teresa Howell
6-Matt Grainger
Parent-Karon Staples
AP-Catherine Waters
AP-Josh Watkins

Mountain Brook Elementary
K-Caroline Christie
K-Kelsey Frey
K-Mitchell Nelson
K-Gretchen Sawyer
K- Julie Summers
1-Katie Beildeman
1-Julie Cox
1-Jamie Jones
1-Joy Palmer
1-Katie Potts
1-Katherine Brown
2-Hannah Garrett
2-Julie Tuck
2-Colleen Varner
3-Catherine Campbell
3-Kim Hall
3-Paulina Haskins

Mountain Brook Elementary Cont'd
3-Tasha Turner
4-Meredith Collins
4-Ashley Margartis
4-Loretta Rowan
4-Jennifer Wilson
5-Alex McCain
6-Lauren Merrill
Parent-Mary Virginia Mandell
AP-Brannon Aaron

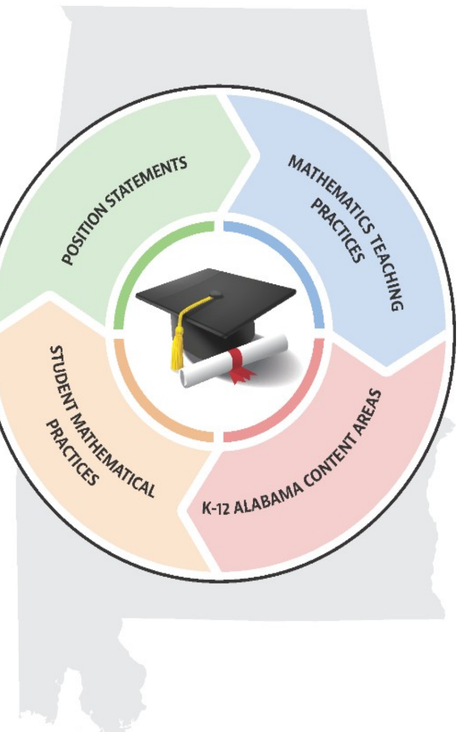
Mtn. Brook Junior High
Madeline Carlton
Madison Gunter
Brittany Henegar
Drew Jackson
Betsy Keller
Cathy Laswell
Lars Porter
Susanna Solar
Stephanie Sorrell
Wendy Spiller
Priscilla Stokes
Parent-Britt Redden
AP-Brook Gibbons

Mtn. Brook High School
Wanda Burns
Morgan Chatham
Jacqueline Cotter
Amy Kathryn Gannon

Mtn. Brook High School Cont'd
Paul Kustos
Jack Letson
Fred Major
Kristina Noto
Christy Stamps
Sara Anne Thomas
Casey Truesdale
Parent-Delia Fischer
AP-Carrie Busby

Math Coaches
Stacy Dorsten
Shannon Millhouse

Central Office
Missy Brooks
Lanie Kent
Holly Martin


POSITION STATEMENTS
MATHEMATICS TEACHING PRACTICES
K-12 ALABAMA CONTENT AREAS
STUDENT MATHEMATICAL PRACTICES

Conceptual Framework

The conceptual framework graphic illustrates the purpose of the *2020 Mountain Brook Curriculum Framework for Mathematics*, which is to ensure that all students receive the mathematics preparation they need to access further educational and professional opportunities, to understand and critique the world around them, and to experience the joy, wonder, and beauty of mathematics (NCTM, 2018). This purpose is depicted by a cyclical pattern formed by position statements, content, student mathematical practices, and mathematics teaching practices that contribute to the development of the mathematically-prepared graduate, represented by the diploma and mortarboard in the center. The cycle has no defined starting or ending point; all of its components must be continuously incorporated into the teaching and learning of mathematics. This integration is essential to the development of an excellent mathematics education program for public schools in Alabama, represented by the shaded map of the state behind the cycle. The four critical components of an excellent mathematics education program are Student Mathematical Practices; Alabama Content Areas and 9-12 Essential Content; Mathematics Teaching Practices; and Position Statements.

The Student Mathematical Practices, also referred to as the Standards for Mathematical Practice, embody the processes and proficiencies in which students should regularly engage as they learn mathematics. These practices include making sense of problems and persevering in solving them; constructing arguments and critiquing the reasoning of others; modeling; using appropriate tools; attending to precision; finding and using structure; and finding and expressing regularity in repeated reasoning. Proficiency in these practices is critical in using mathematics, both within the classroom and in life. Mathematical Practices are a fundamental component in the National Assessment of Educational Progress (NAEP) framework and are included as Alabama standards to be incorporated across all grades.

The vehicle for developing these practices is found in the next component of the cycle, the content standards. These standards specify what students should know and be able to do at the end of each grade or course. The standards are organized in Alabama Content Areas, which are based on the content areas in the 2025 NAEP mathematics framework. They are designed to provide an effective trajectory of learning across the grades that ensures students are well-prepared for future success.

In Grades 9-12, the Alabama Content Standards are organized into subgroups of essential concepts described by the National Council of Teachers of Mathematics (NCTM) in its seminal publication, *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (2018). These essential concepts are designed to be achieved by all students within the first three years of high school mathematics, and they form the foundation for additional coursework designed to meet students' specific post-high school needs and interests.

The next component of the cycle consists of the eight Mathematics Teaching Practices (NCTM, 2014), which should be consistent components of every mathematics lesson across grades K-12. They are:

1. **<u>Establish mathematics goals to focus learning.</u>** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
2. **<u>Implement tasks that promote reasoning and problem-solving.</u>** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving and allow multiple entry points and varied solution strategies.
3. **<u>Use and connect mathematical representations</u>**. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem-solving.
4. **<u>Facilitate meaningful mathematical discourse</u>**. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
5. **<u>Pose purposeful questions</u>**. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense-making about important mathematical ideas and relationships.
6. **<u>Build procedural fluency from conceptual understanding.</u>** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
7. **<u>Support productive struggle in learning mathematics</u>**. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
8. **<u>Elicit and use evidence of student thinking.</u>** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Appendix A provides information that illustrates how these Mathematics Teaching Practices support equitable instruction in the mathematics classroom.

The final component of the cycle is the Position Statements, which are explained in detail in the next section of this document. The Position Statements set forth the foundational requirements for excellence in mathematics education. They deal with access and equity, teaching and learning, curriculum, tools and technology, assessment, and professionalism. All stakeholders, from parents and teachers to policy-makers and business leaders, should examine and embrace these Position Statements to foster excellence in mathematics education in Alabama.

The graphic depicts a dynamic process of establishing and achieving an excellent mathematics program for the State of Alabama. Placement of a diploma at the center is no accident because all efforts are focused on preparing Alabama students for their future. Mathematics will be part of that future, and students must be well equipped to meet the challenges of higher education and meaningful employment.

POSITION STATEMENTS

> *Today, mathematics is at the heart of most innovations in the "information economy," which is increasingly driven by data. Mathematics serves as the foundation for careers in science, technology, engineering, and mathematics (STEM) and, increasingly, as the foundation for careers outside STEM. Moreover, mathematical literacy is needed more than ever to filter, understand, and act on the enormous amount of data and information that we encounter every day. The digital age inundates us with numbers in the form of data, rates, quantities, probabilities, and averages, and this fact of twenty-first-century life increases the importance of and need for today's students to be mathematically and statistically literate consumers, if not producers, of information.* (National Council of Teachers of Mathematics [NCTM], *Catalyzing Change in High School Mathematics*, 2018, p. 1)

Mathematics is critical for the future success of each and every student in Alabama, enabling them to expand their professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics (NCTM, 2018). To help students achieve this goal, schools should implement the six-position statements, which outline foundational practices to ensure excellence in Alabama mathematics programs. Specific pages in *Principles to Action: Ensuring Mathematical Success for All* (NCTM, 2014) that are related to each position statement are indicated in italics at the end of the statement. All stakeholders, from parents and teachers to policy-makers and business leaders, should examine and embrace these principles to foster excellence in mathematics education in Alabama.

Access and Equity in Mathematics Education

An excellent mathematics program promotes access and equity, which requires being responsive to students' backgrounds, experiences, and knowledge when designing, implementing and assessing the effectiveness of a mathematics program so that all students have equitable opportunity to advance their understanding each school year.

Access and equity in mathematics at the school and classroom levels is founded on beliefs and practices that empower each and every student to participate meaningfully in learning mathematics and to achieve outcomes in mathematics that are not predicted by or associated with student characteristics. For all students, mathematics is an intellectually challenging activity that transcends their racial, ethnic, linguistic, gender, and socioeconomic backgrounds. Promoting curiosity and wonder through mathematical discourse is possible when schools and classrooms provide equitable access to challenging curriculum and set high expectations for all students.

Teaching and Learning Mathematics

An excellent mathematics program requires teaching practices that enable students to understand that mathematics is more than finding answers; mathematics requires reasoning and problem-solving in order to solve real-world and mathematical problems.

Teaching matters. Teachers bear the responsibility of ensuring student attainment of content by all who enter their classrooms, regardless of pre-existing skills and knowledge. To increase student proficiency in mathematics, the following mathematics teaching practices (NCTM, 2014) should be integrated into daily instruction:

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem-solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

These mathematics teaching practices are also an element of the Conceptual Framework. See Appendix A.

Student learning involves more than developing discrete mathematical skills; mathematical proficiency has been defined by the National Research Council (2001) to include five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and a productive disposition. Note that procedural fluency involves not just finding answers quickly, but “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (National Research Council, 2001).

Mathematics Curriculum

An excellent mathematics program includes a curriculum that develops the grade-level mathematics content standards along coherent learning progressions which build connections among areas of mathematical study and between mathematics and the real world.

There are differences among standards, resources, and curriculum. The Alabama Content Standards delineate what students are expected to learn within each grade level or course. A curriculum is a sequence of tasks, activities, and assessments that teachers enact to support students in learning the standards while drawing on resources when appropriate. Resources that align with standards are provided for teachers. For example, a standard might read that students will be fluent with two-digit multiplication or that students are fluent with multiplying binomials. A single lesson does not accomplish either of these standards, nor is it productive to have students simply practice this skill in isolation without building from conceptual understanding. A sequence of lessons needs to include examples using concrete models or other appropriate representations to support students in developing strategies that provide a foundation for developing procedural fluency.

Mathematical Tools and Technology

An excellent mathematics program seamlessly integrates tools and technology as essential resources to help students develop a deep understanding of mathematics, communicate about mathematics, foster fluency, and support problem-solving.

Teachers and students should be provided with appropriate tools and technology to support student learning. Students should use mathematical tools and technology in a variety of settings for a variety of purposes. Teachers should design learning activities using tools and technology so that students are mastering concepts, not just practicing skills. For example, base 10 blocks serve as a tool for learning mathematics, and a document camera is a technology that can share the display of base 10 blocks that students are manipulating to acquire conceptual understanding. Interactive technology can help students explore mathematical ideas in order to increase their understanding of mathematics. High school students may manipulate the graph of a function in a computer program using sliders that change the values in its equation in order to better understand particular types of functions. Tools and technology can also be used to differentiate learning experiences. Teachers are provided with professional learning opportunities to support effective student use of these tools and technologies.

Assessment of Mathematics Learning

An excellent mathematics program includes formative assessments to inform future teaching decisions and summative assessment to assess students' ability to problem-solve, to demonstrate conceptual understanding and procedural skills, and to provide feedback to inform students of their progress.

Two types of assessment, summative and formative, require attention in mathematics classrooms. Traditionally, instruction has focused on concluding a learning segment with a variety of summative assessments. While the use of summative assessments is essential, such measures should be used to fully evaluate students' mathematical proficiency, including procedural fluency, problem- solving ability, and conceptual understanding. To expand and improve summative assessment results, students need opportunities throughout the school year to persevere, with teacher support, in struggling with cognitively demanding tasks.

Formative assessment occurs throughout this learning process as students solve tasks and teachers provide support through questioning. During instruction, teachers can learn a great deal about how students think as they draw on prior knowledge to solve novel problems. Formative assessment is possible only when teachers are questioning students' thinking during the learning process. Formative assessments may include the use of questions that drive instructional-decision making, exit slips or bell ringers, teacher observation of student discourse, re-engagement lessons, number talks, evaluations of student work samples, and many other formats. Formative assessment is a powerful tool for making instructional decisions that move student learning forward.

Professional Mathematics Teachers

An excellent mathematics program requires educators to hold themselves and their colleagues accountable for seeking and engaging in professional growth to improve their practice as lifelong learners in order to promote student understanding of mathematics as a meaningful endeavor applicable to everyday life.

Professionals are dedicated to learning and improving their craft, which ultimately benefits students. Mountain Brook Schools is committed to providing opportunities for teachers to embrace learning and professional growth, including providing time and resources for professional learning communities. To achieve growth in the five areas described in this section, districts, schools, and teachers must recognize that continuous professional learning is required. Designing and enacting effective lessons and valid assessments requires teachers to increase their knowledge and skill throughout their careers. To prepare the next generation of thinkers, the mathematics community must work together to support one another in learning. Teaching in ways that promote student collaboration in learning mathematics from and with each other requires adults to model effective collaboration in their own learning and progress.

STUDENT MATHEMATICAL PRACTICES

The Standards for Mathematical Practice, called "Student Mathematical Practices" in this document, describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices are based on important processes and proficiencies that have long-standing importance in mathematics education. The eight Student Mathematical Practices are listed below along with a description of behaviors and performances of mathematically proficient students.

Mathematically proficient students:

1. Make sense of problems and persevere in solving them.

These students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. These students consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculators to obtain the information they need. Mathematically proficient students can explain correspondences among equations, verbal descriptions, tables, and graphs, or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solve complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships. One is the ability to *decontextualize*, to abstract a given situation, represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents. The second is the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

These students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. These students justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments; distinguish correct logic or reasoning from that which is flawed; and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until the middle or upper grades. Later, students learn to determine domains to which an argument applies. Students in all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

These students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, students might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, students might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas and can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and the tools' limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a Web site, and use these to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

These students try to communicate mathematical ideas and concepts precisely. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. Mathematically proficient students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. These students also can pause and reflect for an overview or a shift in perspective. They can observe the complexities of mathematics, such as seeing some algebraic expressions as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that mental picture to realize that the value of the expression cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As students work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details and continually evaluate the reasonableness of their intermediate results.

Connecting the Student Mathematical Practices to the Standards for Mathematical Content

The eight Student Mathematical Practices described on the previous pages indicate ways in which developing student practitioners of the discipline of mathematics increasingly must engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curriculum, assessment, and professional development should be aware of the need to connect the mathematical practices to the mathematical content standards.

The Student Mathematical Practices are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect mathematical practices to mathematical content. Students who lack understanding of a topic may rely too heavily on procedures. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, pause for an overview, or deviate from a known procedure to find a shortcut. Thus, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Student Mathematical Practices and the Standards for Mathematical Content. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus needed to qualitatively improve curriculum, instruction, assessment, professional development, and student achievement in mathematics.