Geometry with Data Analysis Advanced
Overview

While the standards in regular and advanced classes are the same, students in advanced-level classes will have a different learning experience. There is an expectation that comprehension and proficiency will be more profound, and students will study the concepts more in depth. Algebraic topics, like quadratic equations, systems of equations, and simplifying radicals will be incorporated into units of Advanced Geometry. Students will use higher level thinking skills as they explore the content, which will require more abstract thinking. Assessments will be more complex and will require that students organize thoughts, make connections, and analyze more efficiently.

Geometry with Data Analysis is a newly-designed course which builds on the students' experiences in the middle grades. It is the first of three required courses in high school mathematics, providing a common Grade 9 experience for all students entering high-school-level mathematics.

Geometry with Data Analysis builds essential concepts necessary for students to meet their postsecondary goals (whether they pursue additional study or enter the workforce), to function as effective citizens, and to recognize the wonder, joy, and beauty of mathematics (NCTM, 2018). It is important because it develops mathematical knowledge and skills through visual representations prior to the more abstract development of algebra. Beginning high school mathematics with *Geometry with Data Analysis* in Grade 9 offers students the opportunity to build their reasoning and sense-making skills, see the applicability of mathematics, and prepare more effectively for further studies in algebra. The course also focuses on data analysis, which provides students with tools to describe, show, and summarize data in the world around them.

In *Geometry with Data Analysis*, students incorporate knowledge and skills from several mathematics content areas, leading to a deeper understanding of fundamental relationships within the discipline and building a solid foundation for further study. In the content area of Geometry and Measurement, students build on and deepen prior understanding of transformations, congruence, similarity, and coordinate geometry concepts. Informal explorations of transformations provide a foundation for more formal considerations of congruence and similarity, including development of criteria for triangle congruence and similarity. An emphasis on reasoning and proof throughout the content area promotes exploration, conjecture testing, and informal and formal justification. Students extend their middle school work with conjecturing and creating informal arguments to more formal proofs in this course.

In the content area of Algebra and Functions, students perform algebraic calculations with specific application to geometry that build on foundations of algebra from Grades 7 and 8. In the Data Analysis, Statistics, and Probability content area, students build from earlier experiences in analyzing data and creating linear models to focus on univariate quantitative data on the real number line (shape, center, and variability) and bivariate quantitative data on a coordinate plane (creating linear models).

NOTE: Although not all content areas in the grade level have been included in the overview, all standards should be included in instruction.

A focus on mathematical modeling and real-world statistical problem-solving is included across the course; see Appendix E for more information on the modeling cycles for mathematics and statistics. It is essential for students to use technology and other mathematical tools to explore geometric shapes and their properties and to represent and analyze data.

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both within the classroom and in life. **The Student Mathematical Practices are standards to be incorporated across all grades.**

Student Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

The standards indicating what students should know or be able to do at the end of the course are listed in the right columns of the content standard tables. The essential concepts are listed in the left columns. In some cases, focus areas are indicated. Statements in **bold print** indicate the scope of the standard and align the standard to related content taught in other courses. The full scope of every standard should be addressed during instruction.

Geometry with Data Analysis Advanced

Each content standard completes the stem *"Students will…"*

Number and Quantity	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. Extend understanding of irrational and rational numbers by rewriting expressions involving radicals, including addition, subtraction, multiplication, and division, in order to recognize geometric patterns.
Quantitative reasoning includes and mathematical modeling requires attention to units of measurement.	2. Use units as a way to understand problems and to guide the solution of multi-step problems. a. Choose and interpret units consistently in formulas. b. Choose and interpret the scale and the origin in graphs and data displays. c. Define appropriate quantities for the purpose of descriptive modeling. d. Choose a level of accuracy appropriate to limitations of measurements when reporting quantities.

Algebra and Functions	
Focus 1: Algebra	
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	3. Find the coordinates of the vertices of a polygon determined by a set of lines, given their equations, by setting their function rules equal and solving, or by using their graphs.
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *Example*: *Rearrange the formula for the area of a trapezoid to highlight one of the bases.*
Focus 2: Connecting Algebra to Functions	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	5. Verify that the graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which forms a line. 6. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. a. Given the endpoints of the diameter of a circle, use the midpoint formula to find its center and then use the Pythagorean Theorem to find its equation. b. Derive the distance formula from the Pythagorean Theorem.

Data Analysis, Statistics, and Probability	
Focus 1: Quantitative Literacy	
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	7. Use mathematical and statistical reasoning with quantitative data, both univariate data (set of values) and bivariate data (set of pairs of values) that suggest a linear association, in order to draw conclusions and assess risk. *Example*: *Estimate the typical age at which a lung cancer patient is diagnosed, and estimate how the typical age differs depending on the number of cigarettes smoked per day.*
Focus 2: Visualizing and Summarizing Data	
Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to “clean” and organize data, including very large data sets, into a useful and manageable structure – a first step in any analysis of data	8. Use technology to organize data, including very large data sets, into a useful and manageable structure.
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	**9.** Represent the distribution of univariate quantitative data with plots on the real number line, choosing a format (dot plot, histogram, or box plot) most appropriate to the data set, and represent the distribution of bivariate quantitative data with a scatter plot. **Extend from simple cases by hand to more complex cases involving large data sets using technology.** 10. Use statistics appropriate to the shape of the data distribution to compare and contrast two or more data sets, utilizing the mean and median for center and the interquartile range and standard deviation for variability. a. Explain how standard deviation develops from mean absolute deviation. b. Calculate the standard deviation for a data set, using technology where appropriate. 11. Interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers) on mean and standard deviation.

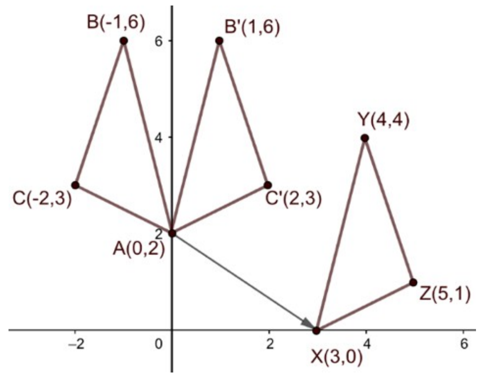
Scatter plots, including plots over time, can reveal patterns, trends, clusters, and gaps that are useful in analyzing the association between two contextual variables.	12. Represent data of two quantitative variables on a scatter plot, and describe how the variables are related. a. Find a linear function for a scatter plot that suggests a linear association and informally assess its fit by plotting and analyzing residuals, including the squares of the residuals, in order to improve its fit. b. Use technology to find the least-squares line of best fit for two quantitative variables.
Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation.	13. Compute (using technology) and interpret the correlation coefficient of a linear relationship. 14. Distinguish between correlation and causation.
Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.	15. Evaluate possible solutions to real-life problems by developing linear models of contextual situations and using them to predict unknown values. a. Use the linear model to solve problems in the context of the given data. b. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the given data.

Geometry and Measurement

Focus 1: Measurement

Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombination.	16. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

	17. Model and solve problems using surface area and volume of solids, including composite solids and solids with portions removed. a. Give an informal argument for the formulas for the surface area and volume of a sphere, cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments. b. Apply geometric concepts to find missing dimensions to solve surface area or volume problems.
Constructing approximations of measurements with different tools, including technology, can support an understanding of measurement.	18. Given the coordinates of the vertices of a polygon, compute its perimeter and area using a variety of methods, including the distance formula and dynamic geometry software, and evaluate the accuracy of the results.
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	19. Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor. 20. Derive and apply the formula for the length of an arc and the formula for the area of a sector.
Focus 2: Transformations	
Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself.	21. Represent transformations and compositions of transformations in the plane (coordinate and otherwise) using tools such as tracing paper and geometry software. a. Describe transformations and compositions of transformations as functions that take points in the plane as inputs and give other points as outputs, using informal and formal notation. b. Compare transformations which preserve distance and angle measure to those that do not. 22. Explore rotations, reflections, and translations using graph paper, tracing paper, and geometry software. a. Given a geometric figure and a rotation, reflection, or translation, draw the image of the transformed figure using graph paper, tracing paper, or geometry software. b. Specify a sequence of rotations, reflections, or translations that will carry a given figure onto another. c. Draw figures with different types of symmetries and describe their attributes.

	23. Develop definitions of rotation, reflection, and translation in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid motions that maps one figure to the other.	24. Define congruence of two figures in terms of rigid motions (a sequence of translations, rotations, and reflections); show that two figures are congruent by finding a sequence of rigid motions that maps one figure to the other. *Example: △ABC is congruent to △XYZ since a reflection followed by a translation maps △ABC onto △XYZ.*  25. Verify criteria for showing triangles are congruent using a sequence of rigid motions that map one triangle to another. a. Verify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. b. Verify that two triangles are congruent if (but not only if) the following groups of corresponding parts are congruent: angle-side-angle (ASA), side-angle-side (SAS), side-side-side (SSS), and angle-angle-side (AAS). *Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show that there must be a sequence of rigid motions will map one onto the other.*

Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other.	26. Verify experimentally the properties of dilations given by a center and a scale factor. a. Verify that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. Verify that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. 27. Given two figures, determine whether they are similar by identifying a similarity transformation (sequence of rigid motions and dilations) that maps one figure to the other. 28. Verify criteria for showing triangles are similar using a similarity transformation (sequence of rigid motions and dilations) that maps one triangle to another. a. Verify that two triangles are similar if and only if corresponding pairs of sides are proportional and corresponding pairs of angles are congruent. b. Verify that two triangles are similar if (but not only if) two pairs of corresponding angles are congruent (AA), the corresponding sides are proportional (SSS), or two pairs of corresponding sides are proportional and the pair of included angles is congruent (SAS). *Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show there must be a set of rigid motions that maps one onto the other.*
Focus 3: Geometric Arguments, Reasoning, and Proof	
Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures.	29. Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. b. Identify different sets of properties necessary to define and construct figures.
Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., two-column, paragraph).	30. Develop and use precise definitions of figures such as angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

	31. Justify whether conjectures are true or false in order to prove theorems and then apply those theorems in solving problems, communicating proofs in a variety of ways, including flow chart, two-column, and paragraph formats. a. Investigate, prove, and apply theorems about lines and angles, including but not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; the points on the perpendicular bisector of a line segment are those equidistant from the segment's endpoints. b. Investigate, prove, and apply theorems about triangles, including but not limited to: the sum of the measures of the interior angles of a triangle is 180˚; the base angles of isosceles triangles are congruent; the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length; a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem using triangle similarity. *c.* Investigate, prove, and apply theorems about parallelograms and other quadrilaterals, including but not limited to both necessary and sufficient conditions for parallelograms and other quadrilaterals, as well as relationships among kinds of quadrilaterals. *Example*: *Prove that rectangles are parallelograms with congruent diagonals.*
Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases one may provide a more accessible or understandable argument than another.	32. Use coordinates to prove simple geometric theorems algebraically. 33. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. *Example*: *Find the equation of a line parallel or perpendicular to a given line that passes through a given point.*

Focus 4: Solving Applied Problems and Modeling in Geometry	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	34. Use congruence and similarity criteria for triangles to solve problems in real-world contexts. 35. Discover and apply relationships in similar right triangles. a. Derive and apply the constant ratios of the sides in special right triangles (45˚-45˚-90˚ and 30˚-60˚-90˚). b. Use similarity to explore and define basic trigonometric ratios, including sine ratio, cosine ratio, and tangent ratio. c. Explain and use the relationship between the sine and cosine of complementary angles. d. Demonstrate the converse of the Pythagorean Theorem. e. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems, including finding areas of regular polygons. 36. Use geometric shapes, their measures, and their properties to model objects and use those models to solve problems. 37. Investigate and apply relationships among inscribed angles, radii, and chords, including but not limited to: the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem-solving.	38. Use the mathematical modeling cycle involving geometric methods to solve design problems. *Examples: Design an object or structure to satisfy physical constraints or minimize cost; work with typographic grid systems based on ratios; apply concepts of density based on area and volume.*