Algebra I with Probability
Overview

Algebra I with Probability is a newly-designed course which builds upon algebraic concepts studied in the middle grades. It provides students with the necessary knowledge of algebra and probability for use in everyday life and in the subsequent study of mathematics. This is one of three courses required for all students. Students can obtain the essential content from this course either by taking the course after completing *Geometry with Data Analysis* in Grade 9 or by completing the middle school accelerated pathway. Alternatively, students who did not take the accelerated pathway in middle school may choose to accelerate in high school by taking *Algebra I with Probability* in Grade 9 along with *Geometry with Data Analysis.*

If students need additional support while taking *Algebra I with Probability*, schools are encouraged to offer a concurrent "lab course" to meet their specific needs. The lab course might review prior knowledge needed for upcoming lessons, reinforce content from previous lessons, or preview upcoming content to ensure that students can fully participate in the required class. Since the lab course does not cover additional mathematical standards, students can receive only an elective credit for each lab course, not a mathematics credit. See further details on the lab courses in the High School Overview. School systems will not offer *Algebra I with Probability* as "A" and "B" courses in which the content is spread over two courses.

Algebra I with Probability builds essential concepts necessary for students to meet their postsecondary goals (whether they pursue additional study or enter the workforce), to function as effective citizens, and to recognize the wonder, joy, and beauty of mathematics (NCTM, 2018). Algebra is important and useful in most careers. It is one of the most common and malleable types of mathematics, because it is valuable in a range of activities from ordinary decision-making to advanced training in scientific and technological fields. The ability to understand and apply algebraic thinking is a crucial stepping stone on a successful journey in life.

Algebra is a collection of unifying concepts that enable one to solve problems flexibly. The study of algebra is inextricably linked to the study of functions, which are fundamental objects in mathematics that model many life situations involving change. This course provides experiences for students to see how mathematics can be used systematically to represent patterns and relationships among numbers and other objects, analyze change, and model everyday events and problems of life and society.

Algebra I with Probability emphasizes functions including linear (as introduced in Grades 7 and 8), absolute value, quadratic, and exponential; and functions as explicit (relation between input and output) and recursive (relation between successive values). Properties of algebra are applied to convert between forms of expressions and to solve equations (factoring, completing the square, rules of powers, and radicals).

Graphing is an important component of study in *Algebra I with Probability*. Graphs of equations and inequalities consist of all points (discrete or continuous) whose ordered pairs satisfy the relationship within the domain and range. Students find points of intersection between two graphed functions that correspond to the solutions of the equations of the two functions, and transform graphs of functions (through translation, reflection, rotation, and dilation) by performing operations on the input or output.

Probability is important because it educates one in the logic of uncertainty and randomness, which occur in almost every aspect of daily life. Therefore, studying probability structures will enhance students' ability to organize information and improve decision-making. The study of probability undergirds the understanding of ratio and proportion in algebra and encourages inferential reasoning about the likelihood of real-life events. Categorical data are represented as marginal and conditional distributions. Parallels are drawn between conditions and events in probability and inputs and outputs of functions.

A focus on mathematical modeling and real-world statistical problem-solving is included across the course. It is essential for students to use technology and other mathematical tools such as graphing calculators, online graphing software, and spreadsheets to explore functions, equations, and probability.

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards to be incorporated across all grades.**

Student Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

The standards indicating what students should know or be able to do are listed in the right columns of the content area tables. The essential concepts are described in the left columns of the content area tables. In some cases, focus areas are indicated within the tables. Only those focus areas which are appropriate for this course are included.

Statements in **bold print** indicate the scope of the standard and align the standard to related content in other courses. The full scope of every standard should be addressed during instruction.

Algebra I with Probability Content Standards

Each content standard completes the stem *"Students will…"*

Number and Quantity	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. Explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for an additional notation for radicals using rational exponents. 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. 3. Define the imaginary number *i* such that $i^2 = -1$.

Algebra and Functions	
Focus 1: Algebra	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	4. Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. *Example: Interpret the accrued amount of investment* $P(1 + r)^t$, *where P is the principal and* r *is the interest rate, as the product of* P *and a factor depending on time* t. 5. Use the structure of an expression to identify ways to rewrite it. *Example: See* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

	6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor quadratic expressions with leading coefficients of one (and leading coefficients with $\|a\| > 1$), and use the factored form to reveal the zeros of the function it defines. b. Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. c. Use the properties of exponents to transform expressions for exponential functions. *Example: Identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* 7. Add, subtract, multiply, and divide polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication. Consider restrictions on the variable(s) when dividing.
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.	8. Explain why extraneous solutions to an equation involving absolute values may arise and how to check to be sure that a candidate solution satisfies an equation.
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	9. Select an appropriate method to solve a quadratic equation in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Explain how the quadratic formula is derived from this form. b. Solve quadratic equations by inspection (such as $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation, and recognize that some solutions may not be real.

	10. Select an appropriate method to solve a system of two linear equations in two variables. a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods.
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	**11.** Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. **Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.** **12.** Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. **Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.** **13.** Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. **Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.**
Focus 2: Connecting Algebra to Functions	
Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics.	14. Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. *Note: The graph of a relation often forms a curve (which could be a line).*

15. Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range.
    a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *Note: If* f *is a function and* x *is an element of its domain, then* f(x) *denotes the output of* f *corresponding to the input* x.
    b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **Limit to linear, quadratic, exponential, and absolute value functions.**

16. Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a function. Explain that a function *f* is a special kind of relation defined by the equation $y = f(x)$.

17. Combine different types of standard functions to write, evaluate, and interpret functions in context. **Limit to linear, quadratic, exponential, and absolute value functions.**
    a. Use arithmetic operations to combine different types of standard functions to write and evaluate functions.
    Example: Given two functions, one representing flow rate of water and the other representing evaporation of that water, combine the two functions to determine the amount of water in a container at a given time.
    b. Use function composition to combine different types of standard functions to write and evaluate functions.
    Example: Given the following relationships, determine what the expression S(T(t)) *represents.*

Function	Input	Output
G	Amount of studying: *s*	Grade in course: *G(s)*
S	Grade in course: *g*	Amount of screen time: *S(g)*
T	Amount of screen time: *t*	Number of followers: *T(t)*

Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	18. Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate. 19. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. *a.* Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. *Note: Include cases where* f(x) *is a linear, quadratic, exponential, or absolute value function and* g(x) *is constant or linear.* 20. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate.
Focus 3: Functions	
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.	**21.** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Extend from linear to quadratic, exponential, absolute value, and general piecewise.** 22. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. a. Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. *Example: A sequence with constant growth will be a linear function, while a sequence with proportional growth will be an exponential function.*
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	23. Identify the effect on the graph of replacing $ff(xx)$ by $ff(xx) + kk, kk \cdot ff(xx)$, $ff(kk \cdot xx)$, and $(xx + kk)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and explain the effects on the graph, using technology as appropriate. **Limit to linear, quadratic, exponential, absolute value, and linear piecewise functions.**

	24. Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. a. Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. b. Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another. c. Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. 25. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). 26. Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. *27.* Interpret the parameters of functions in terms of a context. **Extend from linear functions, written in the form $mx + b$, to exponential functions, written in the form ab^x.** *Example: If the function* $V(t) = 19885(0.75)^t$ *describes the value of a car after it has been owned for* t *years, 19885 represents the purchase price of the car when* $t = 0$*, and 0.75 represents the annual rate at which its value decreases.*
Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	**28.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Note: Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries; and end behavior*. **Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and linear piecewise functions.** **29.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph**. Limit to linear, quadratic, exponential, and absolute value functions.**

	30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph piecewise-defined functions, including step functions and absolute value functions. c. Graph exponential functions, showing intercepts and end behavior.
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	31. Use the mathematical modeling cycle to solve real-world problems involving linear, quadratic, exponential, absolute value, and linear piecewise functions.

Data Analysis, Statistics, and Probability

Focus 1: Quantitative Literacy

Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	32. Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk. *Example: In a clinical trial comparing the effectiveness of flu shots A and B, 21 subjects in treatment group A avoided getting the flu while 29 contracted it. In group B, 12 avoided the flu while 13 contracted it. Discuss which flu shot appears to be more effective in reducing the chances of contracting the flu.* *Possible answer: Even though more people in group A avoided the flu than in group B, the proportion of people avoiding the flu in group B is greater than the proportion in group A, which suggests that treatment B may be more effective in lowering the risk of getting the flu.*

	Contracted Flu	**Did Not Contract Flu**
Flu Shot A	29	21
Flu Shot B	13	12
Total	42	33

Making and defending informed, data-based decisions is a characteristic of a quantitatively literate person.	33. Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. *Example: Investigate whether there appears to be an association between successfully completing a task in a given length of time and listening to music while attempting the task. Randomly assign some students to listen to music while attempting to complete the task and others to complete the task without listening to music. Discuss whether students should listen to music while studying, based on that analysis.*
Focus 2: Visualizing and Summarizing Data	
Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data.	34. Distinguish between quantitative and categorical data and between the techniques that may be used for analyzing data of these two types. *Example: The color of cars is categorical and so is summarized by frequency and proportion for each color category, while the mileage on each car's odometer is quantitative and can be summarized by the mean.*
The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs.	35. Analyze the possible association between two categorical variables. a. Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. b. Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies). c. Identify possible associations and trends in categorical data.

Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts.	36. Generate a two-way categorical table in order to find and evaluate solutions to real-world problems. a. Aggregate data from several groups to find an overall association between two categorical variables. b. Recognize and explore situations where the association between two categorical variables is reversed when a third variable is considered (Simpson's Paradox). *Example: In a certain city, Hospital 1 has a higher fatality rate than Hospital 2. But when considering mildly-injured patients and severely-injured patients as separate groups, Hospital 1 has a lower fatality rate among both groups than Hospital 2, since Hospital 1 is a Level 1 Trauma Center. Thus, Hospital 1 receives most of the severely injured patients who are less likely to survive overall but have a better chance of surviving in Hospital 1 than they would in Hospital 2.*
Focus 3: Statistical Inference (Note: There are no *Algebra I with Probability* standards in Focus 3)	
Focus 4: Probability	
Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities.	37. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). 38. Explain whether two events, A and B, are independent, using two-way tables or tree diagrams.
Conditional probabilities – that is, those probabilities that are "conditioned" by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability.	39. Compute the conditional probability of event A given event B, using two-way tables or tree diagrams. 40. Recognize and describe the concepts of conditional probability and independence in everyday situations and explain them using everyday language. *Example: Contrast the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* 41. Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A, and interpret the answer in context. *Example: the probability of drawing a king from a deck of cards, given that it is a face card, is* $\frac{4/52}{12/52}$, *which is* $\frac{1}{3}$