Algebra II with Statistics
Overview

Algebra II with Statistics is a newly-designed course which builds on the students' experiences in previous mathematics coursework. It is the third of three required courses, and it is to be taken following the successful completion of *Geometry with Data Analysis* and either *Algebra I with Probability* or the middle school accelerated sequence. It is the culmination of the three years of required mathematics content and sets the stage for continued study of topics specific to the student's interests and plans beyond high school.

If students need additional support while taking *Algebra II with Statistics*, schools are encouraged to offer a concurrent "lab course" to meet their specific needs. The lab course might review prior knowledge needed for upcoming lessons, reinforce content from previous lessons, or preview upcoming content to ensure students can fully participate and succeed in the course. Since the lab course does not cover additional mathematical standards, students can receive only an elective credit for each lab course, not a mathematics credit. See further details on the lab courses in the High School Overview.

Algebra II with Statistics builds essential concepts necessary for students to meet their postsecondary goals (whether they pursue additional study or enter the workforce), function as effective citizens, and recognize the wonder, joy, and beauty of mathematics (NCTM, 2018). In particular, it builds foundational knowledge of algebra and functions needed for students to take the specialized courses which follow it. This course also focuses on inferential statistics, which allows students to draw conclusions about populations and cause-and-effect based on random samples and controlled experiments.

In *Algebra II with Statistics,* students incorporate knowledge and skills from several mathematics content areas, leading to a deeper understanding of fundamental relationships within the discipline and building a solid foundation for further study. In the content area of Algebra and Functions, students explore an expanded range of functions, including polynomial, trigonometric (specifically sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. Students also solve equations associated with these classes of functions. In the content area of Data Analysis, Statistics, and Probability, students learn how to make inferences about a population from a random sample drawn from the population and how to analyze cause-and-effect by conducting randomized experiments. Students are introduced to the study of matrices in the Number and Quantity content area.

A focus on mathematical modeling and real-world statistical problem-solving is included across the course. It is essential for students to use technology and other mathematical tools such as graphing calculators, online graphing software, and spreadsheets to explore functions, equations, and analyze data.

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both within the classroom and in life. **The Student Mathematical Practices are standards to be incorporated across all grades.**

Student Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

The standards indicating what students should know or be able to do by the end of the course are listed in the right columns of the content area tables. The essential concepts are described in the left columns of the content area tables. In some cases, focus areas are indicated within the tables. Only those focus areas which are appropriate for this course are included.

Statements in **bold print** indicate the scope of the standard and align the standard to related content in other courses. The full scope of every standard should be addressed during instruction.

Algebra II with Statistics Content Standards

Each content standard completes the stem *"Students will…"*

Number and Quantity	
Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers.	1. Identify numbers written in the form $a + bi$, where a and b are real numbers and $i^2 = -1$, as complex numbers. a. Add, subtract, multiply, and divide complex numbers using the commutative, associative, and distributive properties. b. Represent complex numbers on the complex plane in rectangular form.
Matrices are a useful way to represent information.	2. Use matrices to represent and manipulate data. 3. Multiply matrices by scalars to produce new matrices. 4. Add, subtract, and multiply matrices of appropriate dimensions. 5. Describe the roles that zero and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real numbers. a. Find the additive and multiplicative inverses of square matrices, using technology as appropriate. b. Explain the role of the determinant in determining if a square matrix has a multiplicative inverse.

Algebra and Functions	
Focus 1: Algebra	
Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible.	6. Factor polynomials using common factoring techniques, and use the factored form of a polynomial to reveal the zeros of the function it defines. 7. Prove polynomial identities and use them to describe numerical relationships. *Example: The polynomial identity* $1 - x^n = (1 - x)(1 + x + x^2 + x^3 + \ldots + x^{n-1} + x^n)$ *can be used to find the sum of the first* n *terms of a geometric sequence with common ratio* x *by dividing both sides of the identity by* $(1 - x)$.
Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous.	8. Explain why extraneous solutions to an equation may arise and how to check to be sure that a candidate solution satisfies an equation**. Extend to radical equations.**
The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution.	9. For exponential models, express as a logarithm the solution to $ab^{ct} = d$, where a, c, and d are real numbers and the base b is 2 or 10; evaluate the logarithm using technology to solve an exponential equation. Extend to additional bases.
Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations.	10. Create equations and inequalities in one variable and use them to solve problems**. Extend to equations arising from polynomial, trigonometric (sine, cosine and tangent), logarithmic, radical, and general piecewise functions.** 11. Solve quadratic equations with real coefficients that have complex solutions. 12. Solve simple equations involving exponential, radical, logarithmic, and trigonometric functions using inverse functions.

	13. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales and use them to make predictions. **Extend to polynomial, trigonometric (sine, cosine and tangent), logarithmic, reciprocal, radical, and general piecewise functions.**
Rewrite rational expressions.	14. Add, subtract, multiply, and divide rational expressions.

Focus 2: Connecting Algebra to Functions	
Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology).	15. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. **Extend to cases where *f(x)* and/or *g(x)* are polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.**
Focus 3: Functions	
Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., $f(x) = x^2$), recursive definitions, tables, and graphs.	16. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **Extend to polynomial, trigonometric (sine, cosine and tangent), logarithmic, radical, and general piecewise functions**.
Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family.	17. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(k \cdot x)$, and $(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. **Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.**

Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation.	18. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; and periodicity.* **Extend to polynomial, trigonometric (sine, cosine and tangent), logarithmic, reciprocal, radical, and general piecewise functions.** 19. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **Extend to polynomial, trigonometric (sine, cosine and tangent), logarithmic, reciprocal, radical, and general piecewise functions.** Include compositions of functions. 20. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Extend to polynomial, trigonometric (sine, cosine and tangent), logarithmic, reciprocal, radical, and general piecewise functions.** 21. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **Extend to polynomial, trigonometric (sine, cosine and tangent), logarithmic, reciprocal, radical, and general piecewise functions.** a. Graph polynomial functions expressed symbolically, identifying zeros when suitable factorizations are available, and showing end behavior. b. Graph sine and cosine and tangent functions expressed symbolically, showing period, midline, and amplitude. c. Graph logarithmic functions expressed symbolically, showing intercepts and end behavior. d. Graph reciprocal and other rational functions expressed symbolically, identifying horizontal and vertical asymptotes. e. Graph square root and cube root functions expressed symbolically. f. Compare the graphs of inverse functions and the relationships between their key features, including but not limited to quadratic, square root, exponential, and logarithmic functions.

	22. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle, building on work with non-right triangle trigonometry.
Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems.	23. Use the mathematical modeling cycle to solve real-world problems involving polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions, from the simplification of the problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility.
Build new functions from existing functions.	24. Graph conic sections from second-degree equations, including circles and parabolas.

Data Analysis, Statistics, and Probability	
Focus 1: Quantitative Literacy	
Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks.	25. Use mathematical and statistical reasoning about normal distributions to draw conclusions and assess risk; limit to informal arguments. *Example: If candidate A is leading candidate B by 2% in a poll which has a margin of error of less than 3%, should we be surprised if candidate B wins the election?*
Making and defending informed data-based decisions is a characteristic of a quantitatively literate person.	26. Design and carry out an experiment or survey to answer a question of interest, and write an informal persuasive argument based on the results. *Example: Use the statistical problem-solving cycle to answer the question, "Is there an association between playing a musical instrument and doing well in mathematics?"*

Focus 2: Visualizing and Summarizing Data	
Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable.	27. From a normal distribution, use technology to find the mean and standard deviation and estimate population percentages by applying the empirical rule. a. Use technology to determine if a given set of data is normal by applying the empirical rule. b. Estimate areas under a normal curve to solve problems in context, using calculators, spreadsheets, and tables as appropriate.
Focus 3: Statistical Inference	
Study designs are of three main types: sample survey, experiment, and observational study.	28. Describe the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. *Examples: random assignment in experiments, random selection in surveys and observational studies*
The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups.	29. Distinguish between a statistic and a parameter and use statistical processes to make inferences about population parameters based on statistics from random samples from that population. 30. Describe differences between randomly selecting samples and randomly assigning subjects to experimental treatment groups in terms of inferences drawn regarding a population versus regarding cause and effect. *Example: Data from a group of plants randomly selected from a field allows inference regarding the rest of the plants in the field, while randomly assigning each plant to one of two treatments allows inference regarding differences in the effects of the two treatments. If the plants were both randomly selected and randomly assigned, we can infer that the difference in effects of the two treatments would also be observed when applied to the rest of the plants in the field.*

The scope and validity of statistical inferences are dependent on the role of randomization in the study design.	31. Explain the consequences, due to uncontrolled variables, of non-randomized assignment of subjects to groups in experiments. *Example: Students are studying whether or not listening to music while completing mathematics homework improves their quiz scores. Rather than assigning students to either listen to music or not at random, they simply observe what the students do on their own and find that the music-listening group has a higher mean quiz score. Can they conclude that listening to music while studying is likely to raise the quiz scores of students who do not already listen to music? What other factors may have been responsible for the observed difference in mean quiz scores?*
Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest.	32. Evaluate where bias, including sampling, response, or nonresponse bias, may occur in surveys, and whether results are representative of the population of interest. *Example: Selecting students eating lunch in the cafeteria to participate in a survey may not accurately represent the student body, as students who do not eat in the cafeteria may not be accounted for and may have different opinions, or students may not respond honestly to questions that may be embarrassing, such as how much time they spend on homework.*
The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic.	33. Evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic. a. Simulate a sampling distribution of sample means from a population with a known distribution, observing the effect of the sample size on the variability. b. Demonstrate that the standard deviation of each simulated sampling distribution is the known standard deviation of the population divided by the square root of the sample size.
The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using the margin of error as a measure that describes the sampling variability.	34. Produce a sampling distribution by repeatedly selecting samples of the same size from a given population or from a population simulated by bootstrapping (resampling with replacement from an observed sample). Do initial examples by hand, then use technology to generate a large number of samples. a. Verify that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough. b. Verify that 95% of sample means are within two standard deviations of the sampling distribution from the population mean. c. Create and interpret a 95% confidence interval based on an observed mean from a sampling distribution.

	35. Use data from a randomized experiment to compare two treatments; limit to informal use of simulations to decide if an observed difference in the responses of the two treatment groups is unlikely to have occurred due to randomization alone, thus implying that the difference between the treatment groups is meaningful. *Example: Fifteen students are randomly assigned to a treatment group that listens to music while completing mathematics homework and another 15 are assigned to a control group that does not, and their means on the next quiz are found to be different. To test whether the differences seem significant, all the scores from the two groups are placed on index cards and repeatedly shuffled into two new groups of 15 each, each time recording the difference in the means of the two groups. The differences in means of the treatment and control groups are then compared to the differences in means of the mixed groups to see how likely it is to occur.*

Geometry and Measurement

Focus 1: Measurement	
When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships.	36. Define the radian measure of an angle as the constant of proportionality of the length of an arc it intercepts to the radius of the circle; in particular, it is the length of the arc intercepted on the unit circle.
Focus 2: Transformations (Note: There are no *Algebra II with Statistics* standards in Focus 2)	
Focus 3: Geometric Argument, Reasoning, and Proof (Note: There are no *Algebra II with Statistics* standards in Focus 3)	
Focus 4: Solving Applied Problems and Modeling in Geometry	
Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live.	37. Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline. 38. Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. Extend to additional Pythagorean identities. 39. Derive and apply the formula $A = ½ \cdot ab \cdot sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side, extending the domain of sine to include right and obtuse angles.

	40. Derive and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. **Extend the domain of sine and cosine to include right and obtuse angles.** *Examples: surveying problems, resultant forces*