Applications of Finite Mathematics
Overview

Applications of Finite Mathematics is a newly-designed, specialized course developed for inclusion in the 2019 *Alabama Course of Study: Mathematics. Applications of Finite Mathematics* was developed as a fourth-year course that extends beyond the three years of essential content that is required for all high school students.

Applications of Finite Mathematics provides students with the opportunity to explore mathematics concepts related to discrete mathematics and their application to computer science and other fields. Students who are interested in postsecondary programs of study that do not require calculus (such as elementary and early childhood education, English, history, art, music, and technical and trade certifications) would benefit from choosing *Applications of Finite Mathematic*s as their fourth high school mathematics credit. It may also be a useful supplemental course for students pursuing a career in computer science. This course is an important non-calculus option that presents mathematics as relevant and meaningful in everyday life. Its objective is to help students experience the usefulness of mathematics in solving problems that are frequently encountered in today's complex society.

Finite mathematics includes areas of study that are critical to the fast-paced growth of a technologically advancing world. The wide range of topics in *Applications of Finite Mathematics* includes logic, counting methods, information processing, graph theory, election theory, and fair division, with an emphasis on relevance to real-world problems. Logic includes recognizing and developing logical arguments and using principles of logic to solve problems. Students are encouraged to use a variety of approaches and representations to make sense of advanced counting problems, then develop formulas that can be used to explain patterns. Applications in graph theory allow students to use mathematical structures to represent real world problems and make informed decisions. Election theory and fair division applications also engage students in democratic decision-making so that they recognize the power of mathematics in shaping society.

Applications of Finite Mathematics exhibits tremendous diversity with respect to both content and approach. Teachers are encouraged to engage students using an investigative approach to instruction including the Student Mathematical Practices. Students should be given opportunities to engage in learning and decision-making with technology and hands-on tools.

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both in the classroom and in everyday life. **The Student Mathematical Practices are standards to be incorporated across all grades**

Student Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

The standards in this course extend beyond the essential concepts described in the overview for high school. The standards indicating what students should know or be able to do are listed in the right columns of tables below, organized by relevant content areas.

Applications of Finite Mathematics Content Standards

Each numbered standard completes the sentence stem *"Students will…"*

Logical Reasoning	
The validity of a statement or argument can be determined using the models and language of first order logic.	1. Represent logic statements in words, with symbols, and in truth tables, including conditional, biconditional, converse, inverse, contrapositive, and quantified statements. 2. Represent logic operations such as *and*, *or*, *not*, *nor*, and *x or* (exclusive *or*) in words, with symbols, and in truth tables. 3. Use truth tables to solve application-based logic problems and determine the truth value of simple and compound statements including negations and implications. a. Determine whether statements are equivalent and construct equivalent statements. *Example: Show that the contrapositive of a statement is its logical equivalent.* 4. Determine whether a logical argument is valid or invalid, using laws of logic such as the law of syllogism and the law of detachment. a. Determine whether a logical argument is a tautology or a contradiction. 5. Prove a statement indirectly by proving the contrapositive of the statement.

Advanced Counting	
Complex counting problems can be solved efficiently using a variety of techniques.	6. Use multiple representations and methods for counting objects and developing more efficient counting techniques. *Note: Representations and methods may include tree diagrams, lists, manipulatives, overcounting methods, recursive patterns, and explicit formulas.* 7. Develop and use the Fundamental Counting Principle for counting independent and dependent events. a. Use various counting models (including tree diagrams and lists) to identify the distinguishing factors of a context in which the Fundamental Counting Principle can be applied. *Example: Apply the Fundamental Counting Principle in a context that can be represented by a tree diagram in which there are the same number of branches from each node at each level of the tree.* 8. Using application-based problems, develop formulas for permutations, combinations, and combinations with repetition and compare student-derived formulas to standard representations of the formulas. *Example: If there are* r *objects chosen from* n *objects, then the number of permutations can be found by the product* [n(n-1) … (n-r)(n-r+1)] *as compared to the standard formula* n!/(n-r)!. a. Identify differences between applications of combinations and permutations. b. Using application-based problems, calculate the number of permutations of a set with *n* elements. Calculate the number of permutations of *r* elements taken from a set of *n* elements. c. Using application-based problems, calculate the number of subsets of size *r* that can be chosen from a set of *n* elements, explaining this number as the number of combinations "*n* choose *r*." d. Using application-based problems, calculate the number of combinations with repetitions of *r* elements from a set of *n* elements as "*(n + r – 1)* choose *r*." 9. Use various counting techniques to determine probabilities of events. 10. Use the Pigeonhole Principle to solve counting problems.

Recursion	
Recursion is a method of problem solving where a given relation or routine operation is repeatedly applied.	11. Find patterns in application problems involving series and sequences, and develop recursive and explicit formulas as models to understand and describe sequential change. *Examples*: *fractals, population growth* 12. Determine characteristics of sequences, including the Fibonacci Sequence, the triangular numbers, and pentagonal numbers. *Example: Write a sequence of the first 10 triangular numbers and hypothesize a formula to find the* n*th triangular number.* 13. Use the recursive process and difference equations to create fractals, population growth models, sequences, and series. 14. Use mathematical induction to prove statements involving the positive integers. *Examples: Prove that 3 divides* $2^{2n} - 1$ *for all positive integers* n*; prove that* $1 + 2 + 3 + \ldots + n = n(n + 1)/2$*; prove that a given recursive sequence has a closed form expression.* 15. Develop and apply connections between Pascal's Triangle and combinations.

Networks	
Complex problems can be modeled using vertex and edge graphs and characteristics of the different structures are used to find solutions.	16. Use vertex and edge graphs to model mathematical situations involving networks. a. Identify properties of simple graphs, complete graphs, bipartite graphs, complete bipartite graphs, and trees.

17. Solve problems involving networks through investigation and application of existence and nonexistence of Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. *Note: Real-world contexts modeled by graphs may include roads or communication networks.*
    Example: show why a 5x5 grid has no Hamilton circuit.
    a. Develop optimal solutions of application-based problems using existing and student-created algorithms.
    b. Give an argument for graph properties.
       Example: Explain why a graph has a Euler cycle if and only if the graph is connected and every vertex has even degree. Show that any tree with n *vertices has* n – 1 *edges*.

18. Apply algorithms relating to minimum weight spanning trees, networks, flows, and Steiner trees.
    Example: traveling salesman problem
    a. Use shortest path techniques to find optimal shipping routes.
    b. Show that every connected graph has a minimal spanning tree.
    c. Use Kruskal's Algorithm and Prim's Algorithm to determine the minimal spanning tree of a weighted graph.

19. Use vertex-coloring, edge-coloring, and matching techniques to solve application-based problems involving conflict.
    Examples: Use graph-coloring techniques to color a map of the western states of the United States so that no adjacent states are the same color, determining the minimum number of colors needed and why no fewer colors may be used; use vertex colorings to determine the minimum number of zoo enclosures needed to house ten animals given their cohabitation constraints; use vertex colorings to develop a time table for scenarios such as scheduling club meetings or for housing hazardous chemicals that cannot all be safely stored together in warehouses.

20. Determine the minimum time to complete a project using algorithms to schedule tasks in order, including critical path analysis, the list-processing algorithm, and student-created algorithms.

21. Use the adjacency matrix of a graph to determine the number of walks of length n in a graph.

Fairness and Democracy	
Various methods for determining a winner in a voting system can result in paradoxes or other issues of fairness.	22. Analyze advantages and disadvantages of different types of ballot voting systems. a. Identify impacts of using a preferential ballot voting system and compare it to single candidate voting and other voting systems. b. Analyze the impact of legal and cultural features of political systems on the mathematical aspects of elections. *Examples: mathematical disadvantages of third parties, the cost of run-off elections* 23. Apply a variety of methods for determining a winner using a preferential ballot voting system, including plurality, majority, run-off with majority, sequential run-off with majority, Borda count, pairwise comparison, Condorcet, and approval voting. 24. Identify issues of fairness for different methods of determining a winner using a preferential voting ballot and other voting systems and identify paradoxes that can result. *Example: Arrow's Theorem* 25. Use methods of weighted voting and identify issues of fairness related to weighted voting. *Example: determine the power of voting bodies using the Banzhaf power index* a. Distinguish between weight and power in voting.

Fair Division	
Methods used to solve non-trivial problems of division of objects often reveal issues of fairness.	26. Explain and apply mathematical aspects of fair division, with respect to classic problems of apportionment, cake cutting, and estate division. Include applications in other contexts and modern situations. 27. Identify and apply historic methods of apportionment for voting districts including Hamilton, Jefferson, Adams, Webster, and Huntington-Hill. Identify issues of fairness and paradoxes that may result from methods. *Examples: the Alabama paradox, population paradox*

	28. Use spreadsheets to examine apportionment methods in large problems. *Example: apportion the 435 seats in the U.S. House of Representatives using historically applied methods*

Information Processing	
Effective systems for sending and receiving information include components that impact accuracy, efficiency, and security.	29. Critically analyze issues related to information processing including accuracy, efficiency, and security. 30. Apply ciphers (encryption and decryption algorithms) and cryptosystems for encrypting and decrypting including symmetric-key or public-key systems. a. Use modular arithmetic to apply RSA (Rivest-Shamir-Adleman) public-key cryptosystems. b. Use matrices and their inverses to encode and decode messages. 31. Apply error-detecting codes and error-correcting codes to determine accuracy of information processing. 32. Apply methods of data compression. *Example*: *Huffman codes*

Modeling	
Mathematical modeling and statistical problem-solving are extensive, cyclical processes that can be used to answer significant real-world problems.	33. Use the full Mathematical Modeling Cycle or Statistical Problem-Solving Cycle to answer a real-world problem of particular student interest, incorporating standards from across the course. *Examples: Use a mathematical model to design a three-dimensional structure and determine whether particular design constraints are met; to decide under what conditions the purchase of an electric vehicle will save money; to predict the extent to which the level of the ocean will rise due to the melting polar ice caps; or to interpret the claims of a statistical study regarding the economy.*

Creating Functions to Model Change in the Environment and Society	
Functions can be used to represent general trends in conditions that change over time and to predict future conditions based on present observations.	34. Use elements of the Mathematical Modeling Cycle to make predictions based on measurements that change over time, including motion, growth, decay, and cycling, using polynomial, rational, exponential and logarithmic functions. *Examples: projectile motion, population growth/decay, half-life, mixture problems, water concentration problems.*