Precalculus Advanced
Overview

While the standards in regular and advanced classes are the same, students in advanced-level classes will have a different learning experience. There is an expectation that comprehension and proficiency will be more profound, and students will study concepts in greater breadth and depth, including enrichment and extensions into applications. Students will use higher level thinking skills as they explore the content and more abstract thinking will be necessary. Students in advanced classes are often required to complete more work outside of class than in a regular class. Assessments will be more complex and will require that students organize thoughts, make connections, and analyze more efficiently.

Precalculus is designed for students who intend to pursue a career in science, technology, engineering, or mathematics (STEM) that requires the study of calculus. It prepares students for calculus at the postsecondary level or AP Calculus at the high school level. Students must successfully complete *Algebra II with Statistics* before enrolling in *Precalculus.*

Precalculus builds on the study of algebra and functions in *Algebra II with Statistics*, adding rational functions, all trigonometric functions, and general piecewise-defined functions to the families of functions considered. In addition to focusing on the families of functions, *Precalculus* takes a deeper look at functions as a system, including composition of functions and inverses. *Precalculus* also expands on the study of trigonometry in previous courses and considers vectors and their operations. Other topics, such as statistics, that are frequently added to precalculus courses are not included because the course's primary focus is preparing students for the study of calculus.

In particular, a focus on mathematical modeling is included across the course; see Appendix E for more information on the Mathematical Modeling Cycle. Students' use of technology (such as graphing calculators, online graphing software, and spreadsheets) is essential in exploring the functions and equations addressed in *Precalculus*.

The eight Student Mathematical Practices listed in the chart below represent what students are doing as they learn mathematics. Students should regularly engage in these processes and proficiencies at every level throughout their mathematical studies. Proficiency with these practices is critical in using mathematics, both within the classroom and in life. **The Student Mathematical Practices are standards to be incorporated across all grades.**

Student Mathematical Practices	
1. Make sense of problems and persevere in solving them.	5. Use appropriate tools strategically.
2. Reason abstractly and quantitatively.	6. Attend to precision.
3. Construct viable arguments and critique the reasoning of others.	7. Look for and make use of structure.
4. Model with mathematics.	8. Look for and express regularity in repeated reasoning.

The standards in this course extend beyond the essential concepts described in the overview. The standards indicating what students should know or be able to do are listed in the right columns of the content area tables. Important concepts within these content areas are described in the left columns, and focus areas within the tables are indicated. Only those focus areas which are appropriate for this course are included.

Statements in **bold print** indicate the scope of the standard and align the standard to related content in other courses. The full scope of every standard should be addressed during instruction.

Precalculus Advanced Content Standards

Each numbered standard completes the stem *"Students will…"*

Number and Quantity	
The Complex Number System	
Perform arithmetic operations with complex numbers.	1. Define the constant e in a variety of contexts. *Example: the total interest earned if a 100% annual rate is continuously compounded.* a. Explore the behavior of the function $y=e^x$ and its applications. b. Explore the behavior of $ln(x)$, the logarithmic function with base e, and its applications. 2. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane.	3. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. 4. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *Example*: $(-1+\sqrt{3}i)^3 = 8$ *because* $(-1+\sqrt{3}\bar{i})$ *has modulus 2 and argument 120°.* 5. Use de Moivre's Theorem to compute powers of complex numbers. 6. Reverse-engineer de Moivre's Theorem to find roots of complex numbers. 7. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.	8. Analyze possible zeros for a polynomial function over the complex numbers by applying the Fundamental Theorem of Algebra, using a graph of the function, or factoring with algebraic identities. Use additional techniques of Descartes's Rule of Signs, Intermediate Value Theorem, and bounds on real zeros for further zero analysis.
Limits	
Understand limits of functions.	9. Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. a. Apply limits of functions at specific values and at infinity in problems involving convergence and divergence.
Vector and Matrix Quantities	
Represent and model with vector quantities.	10. Explain that vector quantities have both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. *Examples:* **v**, |**v**|, ||**v**||, v. 11. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. 12. Solve problems involving velocity and other quantities that can be represented by vectors. 13. Find the scalar (dot) product of two vectors as the sum of the products of corresponding components and explain its relationship to the cosine of the angle formed by two vectors. Use dot products to show orthogonality of two vectors.

Perform operations on vectors.	14. Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule, understanding that the magnitude of a sum of two vectors is not always the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Explain vector subtraction, ***v*** – ***w***, as ***v*** + (–***w***), where –***w*** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. 15. Multiply a vector by a scalar. *d.* Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. *Example*: $c(v_x, v_y) = (cv_x, cv_y)$ e. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = \|c\|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\| v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$). 16. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 17. Find cross products of three-dimensional vectors using determinants. 18. Use cross products to find vectors orthogonal to two vectors. Tie cross products to normal vectors of planes and to equations of planes.

Algebra	
Seeing Structure in Expressions	
Write expressions in equivalent forms to solve problems.	19. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems, extending to infinite geometric series. *Examples: calculate mortgage payments; determine the long-term level of medication if a patient takes 50 mg of a medication every 4 hours, while 70% of the medication is filtered out of the patient's blood.*
Arithmetic With Polynomials and Rational Expressions	
Understand the relationship between zeros and factors of polynomials.	20. Derive and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
Use polynomial identities to solve problems.	21. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer, n, where x and y are any numbers.
Rewrite rational expressions.	22. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection; long division; synthetic division, including divisors of degree 2 or more; or, for the more complicated cases, a computer algebra system. 23. Add, subtract, multiply, and divide rational expressions. a. Explain why rational expressions form a system analogous to the rational numbers, which is closed under addition, subtraction, multiplication, and division by a non-zero rational expression. 24. Use partial fraction decomposition to write rational expressions as a sum or difference of simple rational expressions. 25. Use Heaviside Cover-Up Method to achieve partial fraction decomposition for certain rational expressions, and identify situations where Heaviside Cover-Up Method is advantageous/not advantageous.

Reasoning With Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	26. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a clear-cut solution. Construct a viable argument to justify a solution method. **Include equations that may involve linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, and trigonometric functions, and their inverses**. 27. Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.
Solve systems of equations.	28. Represent a system of linear equations as a single matrix equation in a vector variable. 29. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Functions	
Interpreting Functions	
Interpret functions that arise in applications in terms of the context.	30. Compare and contrast families of functions and their representations algebraically, graphically, numerically, and verbally in terms of their key features. *Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; asymptotes; and periodicity.* **Families of functions include but are not limited to linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, trigonometric, and their inverses.** 31. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Extend from polynomial, exponential, logarithmic, and radical to rational and all trigonometric functions.** a. Find the difference quotient $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ of a function and use it to evaluate the average rate of change at a point. b. Explore how the average rate of change of a function over an interval (presented symbolically or as a table) can be used to approximate the instantaneous rate of change at a point as the interval decreases. Tie this concept to derivatives.
Analyze functions using different representations.	32. Graph functions expressed symbolically and show key features of the graph, by hand and using technology. Use the equation of functions to identify key features in order to generate a graph. a. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior. b. Graph trigonometric functions and their inverses, showing period, midline, amplitude, and phase shift.

Building Functions	
Build a function that models a relationship between two quantities.	33. Compose functions. **Extend to polynomial, trigonometric, radical, and rational functions.** *Example: If* T(y) *is the temperature in the atmosphere as a function of height, and* h(t) *is the height of a weather balloon as a function of time, then T(*h(t)*) is the temperature at the location of the weather balloon as a function of time.*
Build new functions from existing functions.	34. Find inverse functions. a. Introduce concept of one-to-one functions. b. Given that a function has an inverse, write an expression for the inverse of the function. *Example*: *Given* $f(x) = 2x^3$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$ *find* $f^{-1}(x)$. c. Verify by composition that one function is the inverse of another. d. Read values of an inverse function from a graph or a table, given that the function has an inverse. e. Produce an invertible function from a non-invertible function by restricting the domain for functions that are not one-to-one. 35. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. **Extend from logarithms with base 2 and 10 to a base of *e*.** Extend to additional bases. 36. Identify the effect on the graph of replacing (x) by $f(x)+k$, $k \cdot f(x)$, $f(k \cdot x)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. **Extend the analysis to include all trigonometric, rational, and general piecewise-defined functions with and without technology.** *Example: Describe the sequence of transformations that will relate* y=sin(x) and y=2sin(3x).

	37. Graph conic sections from second-degree equations, extending from circles and parabolas to ellipses and hyperbolas, using technology to discover patterns. a. Graph conic sections given their standard form. *Example: The graph of* $\frac{x^2}{9}+\frac{(y-3)^2}{4}=1$ *will be an ellipse centered at (0,3) with major axis 3 and minor* axis *2, while the graph of* $\frac{x^2}{9}-\frac{(y-3)^2}{4}=1$ *will be a hyperbola centered at (0,3) with asymptotes with slope ±3/2.* b. Identify the conic section that will be formed, given its equation in general form. *Example*: $5y^2-25x^2=-25$ *will be a hyperbola*. 38. Solve applications involving modeling with equations of conic sections.
Trigonometric Functions	
Recognize attributes of trigonometric functions and solve problems involving trigonometry.	39. Solve application-based problems involving parametric and polar equations. a. Graph parametric and polar equations. b. Convert parametric and polar equations to rectangular form **and vice versa**.
Extend the domain of trigonometric functions using the unit circle.	40. Use special triangles to determine geometrically the values of sine, cosine, and tangent for π/3, π/4, and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π – *x*, π + *x*, and 2π – *x* in terms of their values for *x*, where *x* is any real number. 41. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Model periodic phenomena with trigonometric functions.	42. Demonstrate that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. Tie to one-to-one functions. 43. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.	44. Use trigonometric identities to solve problems. a. Use the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ to derive the other forms of the identity. *Example:* $1 + \cot^2(\theta) = \csc^2(\theta)$ b. Use the angle sum formulas for sine, cosine, and tangent to derive the double angle formulas. c. Use the Pythagorean and double angle identities to prove other simple identities. d. Use double angle identities to prove half-angle identities.