Calculus

The regular Calculus course covers all topics in the AP Calculus AB course except the pace is slower since there is no review for the Advanced Placement exam nor is there a need to complete the course by the Advanced Placement exam date. This course prepares students for their first semester college calculus experience by developing the students' understanding of the concepts of calculus and providing experience with its methods and applications.

The course is taught using a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are important for conceptual understanding. Broad concepts and widely applicable methods are emphasized. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types, although facility with manipulation and computational competence are important outcomes.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions to confirm written work, to implement experimentation, and to assist in interpreting results. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Students will be taught:

Precalculus Review

1. Family of functions – constant, power, polynomial, rational, algebraic, trigonometric, exponential, logarithmic, greatest integer (include domain, range, roots, graphs, periods, symmetry, odd/even, one-to-one, discontinuities, short- and long-term behavior)
    a. Piece-wise functions
    b. Transformations (include translations, stretches, compressions, reflections)
    c. Absolute value in functions
    d. Compositions of functions (include domain and range)
    e. Inverse functions

Limits and Continuity

1. Definition of a limit
    a. Include graphical behavior/discontinuities and the limit proof that corresponds to each behavior/discontinuity
    b. Calculate limits using algebra
    c. One-sided limits
    d. Asymptotic and unbounded behavior (include infinite limits (VA) and limits at infinity (HA, SA, etc.))

   e. Limit theorems
   f. Numerical approximation of limits (estimate limits from graphs or tables of data)
   g. Special limits with extensions including $\lim_{x\to 0}\frac{\sin x}{x}=1$, $\lim_{x\to 0}\frac{\cos x-1}{x}=0$.

2. Introduction to indeterminant forms $(\frac{0}{0},\frac{\infty}{\infty})$
3. Continuity analysis using 3-part definition (include continuity at a point and on an interval)
4. Intermediate Value Theorem

Derivatives

1. Definition of a derivative as the limit of secant line slopes (include both $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)$ and $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=f'(a)$, local linearity) (include instantaneous rate of change as the limit of average rate of change)
2. Determine the derivative as an instantaneous rate of change graphically, numerically, and analytically. (include approximating a rate of change from graphs and tables of values)
3. Determine the relationship between differentiability and continuity
4. Using derivative laws and algebraic computation (23 derivative rules and 2 techniques)
   a. Use the rules of differentiation (sum/difference, product, quotient, chain rule, implicit differentiation, and logarithmic differentiation) to compute derivatives including the derivative of basic function power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
5. Tangent and normal line equations (include linear approximation)
6. Higher-order derivatives
7. Derivatives of inverses (include analytical and graphical proof)
8. Derivative limit templates

Derivative Applications

1. Curve sketching techniques
   a. “If differentiable, then continuous”
   b. Mean Value Theorem for Derivatives (include geometric interpretation)
   c. Rolle’s Theorem (corollary to Mean Value Theorem for Derivatives)
   d. Extreme Value Theorem (include Candidates’ Test) (absolute maximum and minimum)
   e. First Derivative Test and increasing and decreasing functions
   f. Second Derivative Test and concavity
   g. Graphical interpretation of what derivatives (first and second) tell about a function (include major curve pieces and sketches of derivative and antiderivative) (include points at which there are vertical tangents and points at which there are no tangents

   h. Using the second derivative to find extrema
2. The family of *F*, *f*, *f'*, and *f"*
   a. Analyze the derivative as a function and identify the corresponding characteristics of the graphs of the function, derivative, and antiderivative.
   b. Identify corresponding characteristics of graphs of *f* and *f'*
   c. Identify the relationship between the increasing and decreasing behavior of *f* and the sign of *f'*
   d. Identify the relationship between the concavity of *f* and the sign of *f''*
   e. Find points of inflection as places where concavity changes
3. Related rates
4. Optimization
5. L'Hôpital's rule (include $\frac{0}{0}, \frac{\infty}{\infty}$)
   a. Compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth
6. Economics (optional)
7. Motion
   a. Use the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration (include speed increasing and speed decreasing)
8. Translate verbal descriptions into equations involving derivatives and vice versa

Integrals

1. Interpretations of the definite integral as an area accumulator (include numerical integration using geometric methods and left, right, middle Riemann sums, trapezoidal rule, and Simpson's Rule approximations)
2. Antiderivatives and indefinite integrals
3. Integrals of functions (include power, constant multiple, power, sum/difference, chain, trigonometric, exponential, logarithmic, inverse trigonometric)
4. The Fundamental Theorem of Calculus (include Part I (antiderivative) and Part – II (definite integral))
5. Implications of the Fundamental Theorem of Calculus (include graphical accumulation, approximations from tables, net and total change applications)
6. Motion (include both particle and projectile motion) (include position/total distance/displacement, instantaneous/average velocity, speed, acceleration, speed increasing/decreasing)
7. Average value and the Mean Value Theorem for Definite Integrals
8. Connection of the two Mean Value Theorems

Integral Applications

1. Area (include vertical and horizontal slicing AND $A(t) = \int f(t) \cdot dt$)

2. Volume of known cross-sections (include $V(t) = \int A(t) \cdot dt$)
3. Volume of solids of revolution (include discs/washers and cylindrical shells)

Differential Equations

1. Slope fields and antiderivatives (include reading, making, and determining possible graphical solutions for slope fields)

2. Solve Separable Differential Equations using Modeling
   Possible examples: linear, exponential growth/decay and Newton's Law of Heating and Cooling