Mathematics Teaching Practices: Supporting Equitable Mathematics Teaching

Mathematics Teaching Practices	Equitable Teaching
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	● Establish learning progressions that build students' mathematical understanding, increase their confidence, and support their mathematical identities as doers of mathematics. ● Establish high expectations to ensure that each and every student has the opportunity to meet the mathematical goals. ● Establish classroom norms for participation that position each and every student as a competent mathematics thinker. ● Establish classroom environments that promote learning mathematics as just, equitable, and inclusive.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	● Engage students in tasks that provide multiple pathways for success and that require reasoning, problem solving, and modeling, thus enhancing each student's mathematical identity and sense of agency. ● Engage students in tasks that are culturally relevant. ● Engage students in tasks that allow them to draw on their funds of knowledge (i.e., the resources that students bring to the classroom, including their home, cultural, and language experiences).
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and to use as tools for problem solving	● Use multiple representations so that students draw on multiple resources of knowledge to position them as competent. ● Use multiple representations to draw on knowledge and experiences related to the resources that students bring to mathematics (culture, contexts, and experiences). ● Use multiple representations to promote the creation and discussion of unique mathematical representations to position students as mathematically competent.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing students' approaches and arguments.	● Use discourse to elicit students' ideas and strategies and create space for students to interact with peers to value multiple contributions and diminish hierarchical status among students (i.e., perceptions of differences and ability to participate). ● Use discourse to attend to ways in which students position one another as capable or not capable of doing mathematics. ● Make discourse an expected and natural part of mathematical thinking and reasoning, providing students with the space and confidence to ask questions that enhance their own mathematical learning. ● Use discourse as a means to disrupt structures and language that marginalize students.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense-making about important mathematical ideas and relationships.	● Pose purposeful questions and then listen to and understand students' thinking to signal to students that their thinking is valued and makes sense. ● Pose purposeful questions to assign competence to students. Verbally mark students' ideas as interesting, or identify an important aspect of students' strategies to position them as competent. ● Be mindful of the fact that the questions that a teacher asks a student and how the teacher follows up on the student's response can support the student's development of a positive mathematical identity and sense of agency as a thinker and doer of mathematics.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	● Connect conceptual understanding with procedural fluency to help students make sense of the mathematics and develop a positive disposition toward mathematics. ● Connect conceptual understanding with procedural fluency to reduce mathematical anxiety and position students as mathematical knowers and doers. ● Connect conceptual understanding with procedural fluency to provide students with a wider range of options for entering a task and building mathematical meaning.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.	● Allow time for students to engage with mathematical ideas to support perseverance and identity development. ● Hold high expectations, while offering just enough support and scaffolding to facilitate student progress on challenging work, to communicate caring and confidence in students.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	● Elicit student thinking and make use of it during a lesson to send positive messages about students' mathematical identities. ● Make student thinking public, and then choose to elevate a student to a more prominent position in the discussion by identifying his or her idea as worth exploring, to cultivate a positive mathematical identity. ● Promote a classroom culture in which mistakes and errors are viewed as important reasoning opportunities, to encourage a wider range of students to engage in mathematical discussions with their peers and the teacher.

Resources for Grades K – 2

Table 1: Student Mathematical Practices – Kindergarten

1. Make sense of problems and persevere in solving them

Students can…	Because teachers are…
show patience and positive attitudes.	modeling patience and positive attitudes.
ask themselves if their answers make sense.	providing wait-time for processing and finding solutions.
use concrete objects or pictures to help conceptualize and solve a problem.	choosing and posing rich tasks with multiple entry points.
actively engage in problem-solving.	circulating to pose open-ended questions (also including assessing and advancing questions) as they monitor student progress.
understand the approaches of others to solving complex problems.	modeling listening and speaking skills.

2. Reason abstractly and quantitatively

Students can…	Because teachers are…
understand there are multiple ways to break apart the problem in order to find the solution.	asking students to explain their thinking regardless of accuracy.
use symbols, pictures or other representations to describe the different sections of the problem allowing students to use context skills.	accepting varied solutions and representations.
explain their thinking.	facilitating discussion through guided questions and representations.
attend to the meaning of quantities and can use numbers flexibly by applying properties of operations and using objects.	highlighting flexible use of numbers. (Can be done through Math Talks or Number Talks)
ask themselves if their problem-solving and answers make sense.	providing wait-time for processing and finding solutions.

3. Construct viable arguments and critique the reasoning of others

Students can…	Because teachers are…
justify their conclusions, communicate them to others, and respond to the arguments of others.	aiming to create a common mathematical language that can be used to discuss and explain math as well as support or disagree with others' work.
use appropriate math vocabulary while justifying their thinking.	intentionally using math vocabulary that is easily integrated into daily lesson plans in order for students to be able to communicate effectively. (Whole school agreement on math vocabulary.)
listen to the reasoning of others, compare arguments, and ask useful questions to clarify others' thinking.	asking clarifying and probing questions.
make reasonable guesses to explore their ideas.	providing opportunities for students to listen to the conclusions and arguments of others.
	establishing and facilitating a safe environment for rich discussion.
	avoiding giving too much assistance (for example, providing answers, procedures, or too much explanation.)

4. Model with mathematics

Students can…	Because teachers are…
apply the mathematics they know to solve problems arising in everyday life.	choosing real life situations for students because they know math does not end at the classroom door.
reflect on whether the results make sense, possibly improving the model if it does not serve its purpose.	intentionally posing problems connected to previous concepts.
model their thinking with objects, pictures, acting out, numbers, or words.	using purposeful and planned representations.

5. Use appropriate tools strategically

Students can…	Because teachers are…
select and use tools strategically and flexibly, then discuss what worked and what didn't.	choosing open-ended tasks that will require students to select math tools.
detect possible errors by strategically using estimation and other mathematical knowledge.	making appropriate tools available for learning.
use technological tools and resources to solve problems and deepen understanding.	using tools with instruction.
	providing students opportunities to use tools and see significance in real world situations.

6. Attend to precision

Students can…	Because teachers are…
use clear definitions in discussion with others and in their own reasoning. (This includes explaining their thinking using mathematics vocabulary.)	modeling the importance of precision and exact answers in mathematics.
speak and problem-solve, paying attention to exactness and detail.	recognizing and modeling efficient strategies for computation.
give carefully explanations to each other, including when they are confused.	using, and challenging students to use, mathematics vocabulary accurately and consistently.
use appropriate symbols and specify units of measure.	

7. Look for and make use of structure

Students can…	Because teachers are…
use many different skills to determine the answer.	providing time for students to apply and discuss properties.
look closely to discern a pattern or structure.	asking questions about patterns.
adopt mental math strategies based on patterns.	highlighting different mental math strategies through Number Talks, Math Talks, etc.
apply reasonable thoughts about patterns and properties to new situations	

8. Look for and express regularity in repeated reasoning

Students can…	Because teachers are…
work on applying their mathematical reasoning to various situations and problems.	providing tasks with patterns.
continually check their work by asking themselves, “Does this make sense?”	asking about answers before and reasonableness after computations.
look for shortcuts in patterns and repeated calculations	

This project used work created by and adapted from the Departments of Education in Ohio, North Carolina, Georgia, Kansas, and resources created by Achieve the Core, EngageNY, Illustrative Mathematics, NCTM, and Howard County Public School System in Columbia, MD.

Resources for Grades 6 – 8:

TABLE 1: PROPERTIES OF OPERATIONS

Here *a*, *b*, and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.	
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

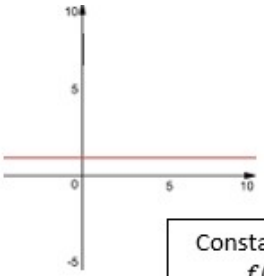
TABLE 2: PROPERTIES OF EQUALITY

Here *a*, *b*, and *c* stand for arbitrary numbers in the rational, real, or complex number systems.	
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

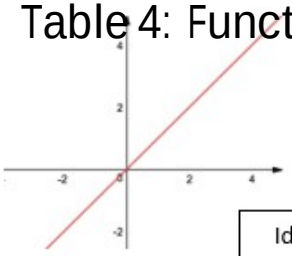
TABLE 3: PROPERTIES OF INEQUALITY

Here *a, b,* and *c* stand for arbitrary numbers in the rational or real number systems.
Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$ then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

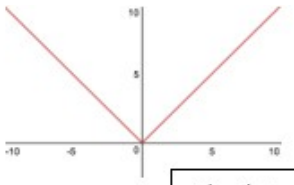
Table 4: Functions



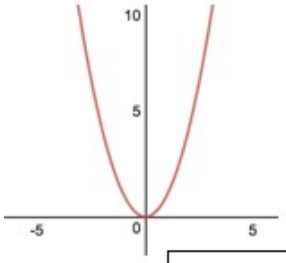
Constant Function
$f(x) = c$



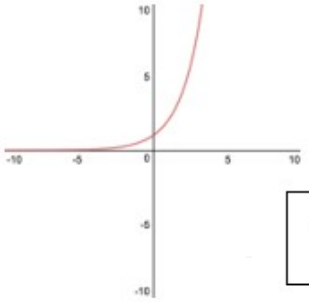
Identity Function
$f(x) = x$



Absolute value Function
$f(x) = |x|$

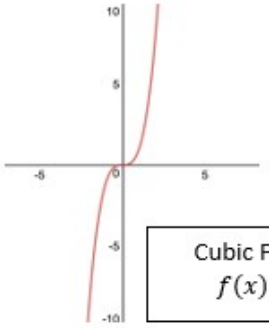


Quadratic Function
$f(x) = x^2$

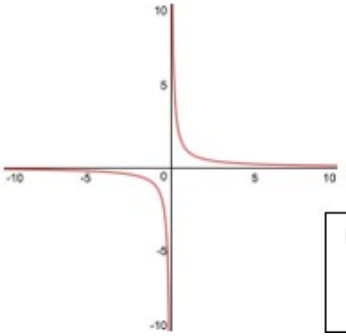


Exponential Function
$f(x) = 2^x$

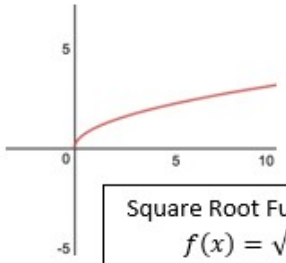
Linear Piecewise Function
$$f(x) = \begin{cases} x; x < 1 \\ 4; 1 \leq x \leq 4 \\ -x + 3; x \geq 4 \end{cases}$$



Cubic Function
$f(x) = x^3$



Reciprocal Function
$f(x) = \frac{1}{x}$



Square Root Function
$f(x) = \sqrt{x}$

Table 5: Reference Page

Some Abbreviations Used in Formulas

bb_1, bb_2 = bases of a trapezoid	C = circumference	*S.A.* = surface area
b = base of a polygon	r = radius	V = volume
h = height or altitude	d = diameter	B = area of a base
l = length	$pi = \pi\pi \approx 3.14$	S = sum of interior angles of a convex polygon
w = width	P = perimeter	n = number of sides of a convex polygon
A = area	m = slope	*L.A.* = lateral area

Formulas

Triangle: $AA = \frac{1}{2}\, bbh$

Parallelogram: $AA = bbh$

Rectangle: $AA = llll$

Trapezoid: $AA = \frac{1}{2}\, h(bb_1 + bb_2)$

Slope: $mm = \frac{yy_2 - yy_1}{xx_2 - xx_1}$

Circle: $AA = \pi\pi rr^2$

$CC = \pi\pi\pi\pi$

$CC = 2\pi\pi rr$

Interest =principal x rate x time

Sum of Measures of Interior Angles of Convex Polygon: $SS = 180(nn - 2)$

Pythagorean Theorem: $cc^2 = aa^2 + bb^2$

Shape	Surface Area	Volume
Rectangular Prism	*L.A.* = Ph *S.A.* = $Ph + 2B$ or *S.A.* = $2(wh + lh + lw)$	$V = Bh$ or $V = lwh$
Cylinder	*L.A.* = $2\pi\pi rh$ *S.A.* = $2\pi\pi rh + 2\pi\pi r^2$	$V = \pi\pi r^2 h$
Square Pyramid	NA	$V = \frac{1}{3}Bh$
Triangular Pyramid	NA	$V = \frac{1}{3}Bh$

Forms of Equations

Standard form of an equation of a line: $Ax + By = C$

Slope-intercept form of an equation of a line: $y = mx + b$

Point-slope form of an equation of a line: $y - y_1 = m(x - x_1)$

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GLOSSARY

Absolute value: The distance from a number to zero.

Absolute value function: See Appendix D, Table 4.

Acute angle: An angle that measures between 0˚ and 90˚.

Addend: Any of the numbers added to find a sum.

Addition and subtraction within 5, 10, 20, 100, or 1000: Addition or subtraction of two whole numbers with whole number answers and with sum or minuend in the range 0-5, 0-10, 0-20, 0-100 or 0-1000, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses: Two numbers whose sum is 0 are additive inverses of one another.
Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + -\frac{3}{4} = 0$

Adjacent angles: Two angles that share a common vertex and a common side but do not share any interior points.

Algorithm: A process or set of rules for solving a problem**.**

Amplitude: The distance from the midline to the maximum or minimum value of a periodic function, calculated as (maximum value - minimum value)/2**.**

Arc: A section of a circle contained between two points.

Area: The measure of the interior of a two-dimensional figure (square units).

Area model: A concrete model for multiplication or division made up of a rectangle. The length and width represent the factors and the area represents the product. Area models can also be used for multiplying and factoring polynomials and for completing the square.

Arithmetic sequence: A sequence in which the difference between two consecutive terms is constant.

Array: A concrete model for multiplication in which items are arranged in rows and columns. Each row (or column) represents the number of groups and each column (or row) represents the number of items in a group. Example: The array shown below represents 5 x 4 = 20, since there are 5 rows of 4 stars for a total of 20 stars. It could also represent 4 x 5 = 20, since there are 4 columns of 5 stars for a total of 20 stars.

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Association (Data Analysis, Statistics, and Probability, Grades 9-12): A relationship between two categorical variables in which a specific value of one variable is more likely to coincide with a specific value of another variable.

Asymptote: A line that a curve becomes arbitrarily close to as one of the coordinates of the curve approaches infinity.

Automaticity: The ability to perform mathematical operations accurately and quickly.

Average: See *mean*.

Axis of symmetry: A line that divides a function into two congruent parts so that points on one side of the line are a reflection of the points on the other side; for all values of x, $f(x) = f(-x)$.

B (as in V=Bh): Area of the base of a three-dimensional figure.

Benchmark number: A number or numbers that help to estimate a value. Examples: 10, 100, 0, ½, and 1.

Bias (statistical bias): Using a sampling method that favors some outcomes over others so that it consistently overestimates or underestimates the true value.

Bivariate data: Set of paired values for two related variables. Examples: a person's race and gender (both categorical), a person's height and weight (both quantitative), or a person's gender and height (categorical and quantitative).

Box-and-whisker plot (box plot): A method of visually displaying variation in a set of data values by using the median, quartiles, and extremes of the data set.

Categorical data: Variables with values that may be divided into groups. Examples: race, gender, educational level, zip code.

Categorical variable: A variable whose possible values can be placed into groups and for which arithmetic (e.g., average) does not make sense.

Causation: A change in one variable results in the change of another variable.

Cavalieri's Principle: A method for showing that two solids have the same volume by showing that areas of corresponding cross sections are equal.

Center: A value that represents a typical value or middle of a set of quantitative data, such as mean or median.

Center of a circle: A point that is equidistant from all points on a circle in a plane.

Chord: A segment joining two points on a circle.

Circle: A set of points in a plane equidistant from a given point, which is called the center.

Circumscribed polygon: A polygon whose sides are all tangent to a given circle. The circle is said to be **inscribed** in the polygon.

Closure: If an operation is performed on two elements of a set, the result is always an element of a set.

Coefficient: The numerical factor in a term that contains one or more variables. Example: In $2xy^2$, 2 is the coefficient.

Combination: A way of selecting items from a set or collection, such that the order of selection does not matter**.**

Complementary angles: Two angles whose measures have a sum of 90˚.

Complex fraction: A fraction A/B where A and/or B are fractions (where B is non-zero).

Complex number: A number written in the form $a + bi$, where a and b are real numbers and b is multiplied by the imaginary unit i.

Compose: To put together a number or shape using existing numbers or shapes.

Composition: The process of combining two functions in which one function is performed first and then its output is used as the input for the second function.

Conditional probability: The probability of an event given that another event has occurred.

Conditional relative frequency: The ratio of a joint relative frequency to a related marginal relative frequency.

Confidence interval: An interval combined with a probability statement that is used to express the degree of uncertainty associated with a sample statistic and that estimates where a population parameter will lie. Example: We are 95% confident that the mean salary of the population lies within $512 of the sample mean.

Congruent figures: Two plane or solid figures are congruent if one can be mapped to the other by a rigid motion (a sequence of rotations, reflections, and translations).

Conic section: A figure formed by the intersection of a plane and a right circular cone. Examples: ellipse, parabola, or hyperbola.

Conjugate: An expression formed by changing the sign of the second term. For numbers of the form $a + bi$ where a and b are real numbers, the conjugate is $a - bi$, such that the product of the number and its conjugate is a^2+b^2.

Constant: A variable that has a fixed value.

Constant function: A function that has the same output for every input. See Appendix D, Table 4.

Constraint: A restriction on what solutions to a problem are valid.

Continuous quantitative data: Data items within a set that can take on any value within a range, including non-integer values. Example: A person's height or the length of a person's foot.

Coordinate plane (system): A two-dimensional system for locating points in the plane consisting of two number lines, where the horizontal number line (x-axis) and vertical number line (y-axis) are perpendicular and intersect at a point (origin). Points are described by their relative locations on the two number lines.

Correlation: An association or relationship between two quantitative variables.

Correlation coefficient: A statistic (r) which measures the strength of the linear association between two quantitative variables. Values range from -1 to 1, where 1 denotes a perfect positive relationship, -1 a perfect negative relationship, and 0 no relationship at all.

Corresponding parts: The sides, angles, and vertices of one figure that are mapped onto those of another figure using a geometric transformation.

Counting on: A strategy for finding the number of objects in a group without having to count every member of the group. Example: If a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the entire stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Cube root: A cube root of x is the number that, when multiplied by itself three times (or cubed), gives the number x. Example; 2 is the cube root of 8 because $2^3 = 2 \cdot 2 \cdot 2 = 8$.

Cubic function: A polynomial function whose highest degree is 3. See Appendix D, Table 4.

Decimal number: A quantity represented using base-10 notation, using a decimal point to separate the whole number and fractional parts.

Decompose: To separate numbers or shapes into component or smaller parts.

Denominator: The divisor in a fraction or rational expression.

Dependent events: Two or more events in which the outcome of one is affected by the outcome(s) of the other(s).

Dependent variable: A variable in an expression, equation, or function whose value is determined by the choice of the other variables.

Descriptive statistics: Values used to describe features of a univariate or bivariate quantitative data set. Common statistics involve measures of center and measures of spread.

Dilation: A geometric transformation in which the image of each point lies along the line from a fixed center point through the given point, where its distance is multiplied by a common scale factor. Images of geometric figures using a dilation are similar to the given figures.

Discrete quantitative data: Data items within a set which can take on only a finite number of values. Examples: A person's shoe size or rolling a die.

Distribution: A description of the relative number of times each possible outcome of a statistical variable occurs or will occur in a number of trials.

Dividend: a where $a \div b = c$.

Divisor: b where $a \div b = c$.

Domain: The set of inputs for a function or relation.

Dot plot: See *line plot.*

Ellipse: The set of all points in a two-dimensional plane where the sum of the distance from two distinct points (foci) is constant.

Empirical Rule: A description of a normal distribution of data. In a normal distribution, almost all data will fall within three standard deviations of the mean. Approximately 68% of the values lie within one standard deviation of the mean, approximately 95% of the values lie within two standard deviations of the mean, and approximately 99.7% lie within three standard deviations of the mean. Also known as the 68-95-99.7% Rule.

End behavior: In a function, the values the dependent variable approaches as the independent variable approaches either negative or positive infinity.

Equation: A mathematical relationship in which two expressions are equal.

Equivalent expression: Expressions that represent the same amount; equations or inequalities that have the same solution set.

Evaluate: To determine or calculate the value of an expression once specific values have been substituted for each of the variables in the expression.

Even function: A function whose graph is symmetric with respect to the *y*-axis. $f(x) = f(-x)$ for all x.

Event: A set of possible outcomes of an experiment or study; a subset of the sample space.

Expanded form: A multi-digit number written as a sum of single-digit multiples of powers of ten. Example: 643 = 600 + 40 + 3.

Expected value: For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

Experiment: A controlled study used to find the differences between groups subjected to different treatments.

Experimental probability: The number of occurrences of an event in a set of trials divided by the total number of trials. Contrast to *theoretical probability*.

Exponent: The small number placed to the upper right of a base number indicating how many copies of the base number are multiplied together. Example: In 2^4 , 2 is the base number, and 4 is the exponent.

Exponential function: A function in which the dependent variable is an exponent, with a constant base. See Appendix D, Table 4.

Exponential notation: A general version of scientific notation in which the base does not have to be 10.

Expression: A mathematical phrase that combines numbers and/or variables using mathematical operations. Examples: 3 x 6; 4 + 7 x 3; 8; $2h + 3kk$

Extraneous solution: A solution to an equation that emerges from the process of solving the problem but is not a valid solution to the original problem.

Factors of a number or expression: Numbers or terms multiplied together to find a product. Example: a and b, where $a \times b = c$. Where c is a whole number, integer, or polynomial, a and b are often limited to whole numbers, integers, or polynomials respectively. c is called the **product**.

Fair share model (partitive division): A division model in which the total number of items and the number of groups is known and the number of items in each group is the unknown.

Fibonacci Sequence: A recurrence relation where successive terms are found by adding the two previous terms given that the first two terms are 0 and 1.

Fluency: The ability to use strategies and/or procedures that are flexible, efficient, accurate, and generalizable to answer mathematical questions.

Fraction: A number expressible in the form $\frac{a}{b}$ where a is a whole number and b is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) See also *rational number*.

Frequency: The number of cycles of a periodic function that occur within a given distance of the dependent variable; the number of occurrences of a value in a distribution of discrete data.

Frequency table: A table that lists the frequency of each item in a distribution of discrete data. In the lower grades, tally marks may be used to record the number of times each item occurs.

Function: Relationship between two variables where each input (value of the independent variable) has a single output (value of the dependent variable).

Function notation: Use of *f(x)* notation to define a function.

Fundamental Counting Principle: A way to figure out the total number of ways different events can occur (outcomes) in a probability problem. If there are two independent events with m and n possible outcomes, respectively, then there are $mm \cdot nn$ total possible outcomes for the two events together.

Geometric sequence: A sequence in which the ratio between two consecutive terms is constant.

Geometric series: The sums of the terms in a geometric sequence.

Geometric transformation: A function that maps all the points of the plane onto the plane.

Greatest integer function: A function that assigns each input value to the greatest integer that is less than that value. See Appendix D, Table 4.

Histogram: A graphical display representing the frequency distribution of continuous numerical data where the data are grouped into bins. Each bin represents a range of data.

Hyperbola: The set of all points in a two-dimensional plane where the difference of the distances from two distinct points (foci) is constant.

Identity: An equation that is true for all values of the variables.

Identity function: A function that assigns each input value to itself. See Appendix D, Table 4.

Identity matrix: A square matrix whose elements consist of ones along the diagonal (from upper right to lower left) and zeros everywhere else. Multiplying any matrix by an identity matrix of the appropriate size will yield that same matrix.

Identity property of 0: $a + 0 = 0 + a = a$, for all a. See Appendix D, Table 1.

Image: A figure or set of points that results from a transformation.

Imaginary number (complex number): The square root of a negative number. The square root of negative 1 or $\sqrt{(-1)}$ is defined to be i. See also *complex number.*

Independence: A lack of association between two categorical variables, in which a specific value of one variable is equally likely to coincide with all values of the other variable.

Independent events: Two or more events in which the outcome of one is not affected by the outcome of the other(s).

Independent variable: A variable in an expression, equation, or function whose value is freely chosen regardless of the value of the other quantities.

Inequality: A mathematical sentence that compares the order of two quantities: greater than ($>$), greater than or equal to ($\geq$), less than ($<$), less than or equal to ($\leq$).

Inference (statistical inference): Conclusions drawn, based on data.

Inferential statistics: The mathematical science of using data collection and analysis to make predictions about a population based on a random sample or to draw conclusions of cause and effect based on a random assignment of treatments.

Inscribed polygon: A polygon whose vertices are all contained on a given circle. The circle is said to be **circumscribed** about the polygon.

Integer: Any of the natural numbers, the negatives of these numbers, or zero.

Interquartile range: A measure of variation in a set of numerical data. The interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9.

Inverse function: A function that "undoes" a given function, mapping that function's outputs to its inputs.

Inverse operations: Operations that "undo" one another. Example: 5+4=9 and 9-4=5, demonstrating that subtraction is the inverse operation of addition.

Irrational number: A real number r such that there are no integers a and b ($b \neq 0$) where $r = a \div b$ (such as $\pi\pi$).

Joint relative frequency: The ratio of the frequency in a particular category and the total number of data values.

Like terms: Two or more terms that have the same variables and powers, but possibly different coefficients. Note: any constant, c, can be written as $cc \cdot xx^0$. Thus, all constants are like terms.

Limit: A value that a function approaches as the input approaches some number.

Line plot: A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line (also known as a *dot plot*).

Line segment: Set of points between two endpoints, A and B, that lie on the line that contains A and B.

Linear association: A relationship between two quantitative variables that can be represented using a linear equation. On a scatter plot, the relationship can be represented by a line.

Linear equation: An equation of two linear expressions. Linear equations including two variables can be represented by a line in the coordinate plane.

Linear expression: An expression whose terms each include at most one variable with degree one (raised to the first power). For example, $2x + 7$ or $3x + 4y – 11$.

Linear function: A function whose output is determined by a linear expression. A linear function can be represented by a line in the coordinate plane. See Appendix D, Table 4.

Logarithm: The exponent to which a given base must be raised to yield a given value. Example: the logarithm of 1000 base 10 is 3, since $10^3=1000$.

Logarithmic function: The inverse function of an exponential function.

Magnitude of a vector: The length of a vector.

Marginal relative frequency: The ratio of the sum of the joint relative frequency in a row or column and the total number of data values.

Mass: A measure of how much matter is in an object, usually defined in grams.

Mathematical modeling: Using mathematics to solve a complicated real-world problem where there is no clear-cut method to solve the problem. Note that mathematical modeling is different from using manipulatives and other representations to model mathematical concepts. See Appendix E.

Matrix: A rectangular array of numbers or other data. The dimensions of the matrix are determined by its number of rows and columns. Example: The dimensions of a matrix with two rows and three columns would be 2 x 3.

Maximum: The greatest value in a data set or the greatest possible value of an expression or function.

Mean (arithmetic mean or average): A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list; the balance point. For the data set $\{x_1, x_2, x_3, \ldots, x_n\}$, the mean (often written $\bar{x}$) = $\frac{1}{n}\sum_{i=1}^{n} x_i$. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation: The average distance between each value in a set of numerical data and the mean of the data set. For the data set $\{x_1, x_2, x_3, \ldots, x_n\}$ with mean $\bar{x}$ MAD = $\frac{1}{n}\sum_{i=1}^{n} |x_i - \bar{x}|$. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 19.96.

Median: A measure of center in a set of numerical data. The median of a list of values is the number at the center of an ordered list—or the mean of the two numbers in the middle if there are an even number of elements. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Metric system: A measurement system used throughout the world that is based on units that are related by powers of 10, using a standard set of prefixes. The base unit for measuring length is the meter, the base unit for capacity is the liter, and the base unit for mass is grams. Commonly-used prefixes include *milli-* denoting 1/1000 of the base unit, *centi-* denoting 1/100 of a unit, and *kilo-* denoting 1000 base units. For example, a milliliter is 1/1000 of a liter, a centimeter is 1/100 of a meter, and a kilometer is 1000 meters. Temperature is measured in degrees Celsius, in which 0˚ C. is

the freezing point of water and 100˚ is the boiling point of water.

Midline: In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Midpoint: A point on a segment that is equidistant from the endpoints.

Minimum: The smallest value in a data set or the smallest possible value of an expression or function.

Mode: The value that occurs most frequently in a data set.

Modular arithmetic: If *a* and *b* are integers and *m* is a positive integer, then *a* is said to be congruent to *b modulo m* if *m* divides *a – b.*

Monomial: A mathematical expression consisting of a single term.

Multiple: A number that is the result of multiplying a given whole number (or integer) by another whole number (or integer). Example: Multiples of 5 are 0, 5, 10, 15, 20, 25, 30….

Multiplication: A mathematical operation involving two factors. One factor describes the number of groups or sets, the other factor describes the number of items in a group or set and the result, or product, describes the total number of items.

Multiplication and division within 100: Multiplication or division of two whole numbers with whole number answers, with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses: Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

Natural number: Whole numbers excluding zero; “counting numbers.”

Nonresponse bias: Bias that occurs when individuals chosen for a sample are unable or unwilling to respond and differ in meaningful ways from those who do respond.

Normal distribution: A naturally occurring distribution that is symmetric about the mean, bell shaped, and dispersed systematically. Also known as a *normal curve*.

Number line diagram: A line diagram used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Numerator: The number in a fraction that indicates the number of parts of the whole that are being considered; the top number in a fraction.

Obtuse angle: An angle measuring more than 90˚ but less than 180˚.

Odd function: A function whose graph is 180˚ rotationally symmetric about the origin. $f(-x) = -f(x)$ for all x.

Open figure: A shape made up of line segments with at least one line segment that isn't connected to anything at one of its endpoints.

Ordered pair: A set of numbers (x, y) where the first number (x-coordinate) shows position to the left or right of the origin (0, 0) and the second number (y-coordinate) shows position above or below the origin on a coordinate plane.

Outlier: Values in a data set that are much smaller or larger than the rest of the values.

Parabola: The set of all values in a two-dimensional plane that are the same distance from a fixed point (focus) and a line (directrix).

Parallel lines: Two lines in a plane that do not intersect.

Parameter: In statistics, a numerical measure that describes a characteristic of a population. In algebra, parameters are quantities that influence the output or behavior of a mathematical object that are not explicitly varied but viewed as being held constant. Example: for $yy = mmxx + bb$, m and b are parameters and x and y are variables. Also, the independent variable used in parametric equations.

Parametric equations: A set of equations used to define the coordinates of a set of points in terms of an independent variable called a parameter.

Partial product: A part of the product in a multiplication calculation, usually based on place value and the distributive property.

Partial quotient: A part of the quotient in a division calculation, usually based on place value and the distributive property.

Pattern: Set of numbers or objects that can be described by a specific rule.

Percent rate of change: A rate of change expressed as a percent. Example: If a population grows from 50 to 55 in a year, it grew by $\frac{5}{50} = 10\%$ per year.

Perimeter: The distance around a figure or object.

Period: The length of the dependent variable in a periodic function for a complete cycle to occur.

Permutation: An arrangement of all the members of a set or collection into some sequence or order, or if the set is already ordered, the reordering of its elements, a process called permuting.

Perpendicular lines: Two lines or line segments that intersect to form a right angle (90˚).

Piecewise function: A function in which more than one formula is used to define the output over different intervals (pieces) of the domain.

Pigeonhole Principle: A concept used in problem-solving, which states that if there are more pigeons than pigeonholes, then at least one pigeonhole has at least two pigeons in it.

Plane figure: A two-dimensional shape.

Point: An exact position or location on a plane surface or in space.

Polar equations: An equation defining an algebraic curve expressed in polar coordinates r and θ, where r is the distance from the origin and θ is the angle of rotation from the x-axis.

Polygon: Any closed plane shape formed by line segments (also called **sides** of the polygon,) where each endpoint of a side (also called a **vertex** of the polygon) is shared by exactly two sides.

Polyhedron: A three-dimensional figure formed by polygons (also called **faces** of the polyhedron,) where each side of a face (also called an **edge** of the polyhedron) is common to exactly two faces. The vertices of the polygons (also called **vertices** of the polyhedron) may be shared by multiple edges.

Polynomial: A mathematical expression containing real numbers and variables related only by the operations of addition, subtraction, multiplication, and non-negative integer exponents. The standard form of a polynomial is written as a sum of terms, each of which is simplified to be the product of a constant and/or variables raised to whole number exponents greater than 0. Example: $3x^2 + 4xy^3 + 9$ is a polynomial written in standard form.

Population: The entire group of objects, people, or events about which information is sought in a statistical study.

Population distribution: The distribution of all values of a variable for all individuals in the population.

Power: Exponent; the number of times a base number is multiplied by itself.

Preimage: A figure or set of points that is an input to a transformation.

Prime number: A number that has exactly two factors, 1 and itself.

Prism: Polyhedron with two congruent and parallel faces that are polygons; the rest of the faces are parallelograms.

Probability: A number between 0 and 1 used to quantify likelihood.

Probability distribution: The set of possible values of a random variable with a probability assigned to each.

Probability model: A model used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also *uniform probability model*.

Product: The result when two or more numbers or terms are multiplied together.

Properties: Characteristics, such as color, size, or height; statements that are always true for some class of objects.

Proportional: Term describing two quantities that are related by a constant ratio, whose value is called the constant of proportionality.

Pythagorean Theorem: The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse. $aa^2 + bb^2 = cc^2$, where *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse.

Quadrant: One of the four parts into which a coordinate plane is divided by the *x*-axis and *y*-axis.

Quadratic equation: An equation which equates a quadratic expression to another quadratic expression, linear expression, or constant.

Quadratic expression: A polynomial expression in one variable where the largest exponent of the variable is 2 when written in standard form.

Quadratic formula: $x = \frac{-bb \pm \sqrt{bb^2 - 4aaaa}}{2aa}$, where *x* is the solution of an equation of the form $ax^2 + bx + c = 0$ and $aa \neq 0$.

Quadratic function: A function in which each input value is related to its output value by a quadratic expression. See Appendix D, Table 4.

Qualitative variable: See *categorical variable.*

Quantitative literacy: The ability to use mathematical and statistical reasoning to address practical, civic, professional, recreational, and cultural issues.

Quantitative variable: A variable whose possible values are numerical and represent a measurable quantity. Examples: a person's weight or age.

Quartile: A grouping of data points. Three points in a ranked set of data divide the data into four equal groups, where each group contains a quarter of the data points. The three points include the median of the full set of data (Q2), the median of the set of values above the median of the full set (Q3), and the median of the set of values below the median of the full set (Q1).

Quotient: *c* where $a \div b = c$.

Radian: The radian measure of an angle is equal to the ratio of the length of the subtended arc of the angle to the radius.

Radical equation: An equation in which at least one variable expression is under a radical.

Random sample: A method of selection in which members of the statistical population are chosen by chance, with each member of the population having an equal probability of being chosen.

Random variable: A statistical variable whose values are the result of a random process.

Range: The maximum value minus the minimum value in a data set. The range of a function is the set of outputs from the domain.

Ratio: The multiplicative comparison of two non-zero quantities. Represented as *a:b, a/b,* and *a* to *b.*

Rational expression: A quotient of two polynomials with a non-zero denominator.

Rational number: A number that can be expressed as a fraction in the form of $\frac{aa}{bb}$ where $bb \neq 0$.

Ray: A set of points on a line that begins at a point (called the endpoint) and extends infinitely in one direction.

Real number: The set of all possible values on a number line; that is, the set of all rational and irrational numbers. Each real number can be represented by a decimal number, either finite or infinite.

Reciprocal: The multiplicative inverse of a number.

Reciprocal function: A function defined by the reciprocal of a linear function. See Appendix D, Table 4.

Remainder: Amount remaining when one whole number is unevenly divided by another whole number.

Repeating decimal: A decimal number in which a string of one or more digits following the decimal point repeats indefinitely. Example: 2.137232323… is a repeating decimal since the string “23” repeats.

Response bias: Bias that results from problems in the measurement process. Examples: leading questions, social desirability.

Right angle: An angle that measures exactly 90˚.

Rigid motion: A geometric transformation of points consisting of a sequence of one or more translations, reflections, and/or rotations which preserve distances and angle measures.

Round: To use mathematical rules to alter a number to one that is less exact but easier to use in mental computation. The number is kept close to its original value.

Sample: A subset of data selected from a statistical population by a defined procedure.

Sample space: All possible outcomes of an event.

Sample survey: A study that obtains data from a subset of a population to estimate population parameters.

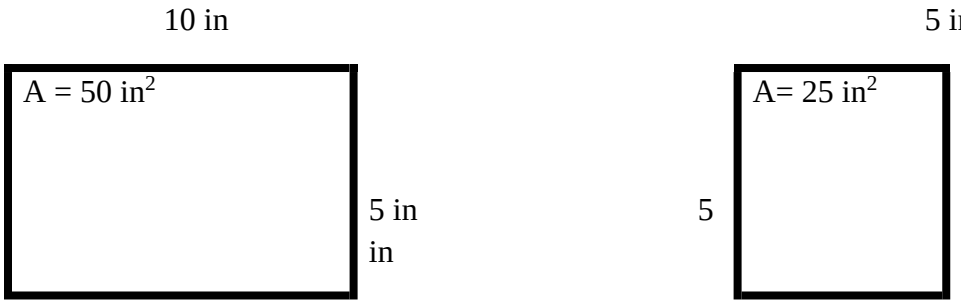
Sampling bias: The partiality that occurs when a sample statistic does not accurately reflect the true value of the parameter in the population.

Sampling distribution: A distribution of values taken by a statistic in all possible samples of the same size from the same population.

Scalar: Numerical values or quantities that are fully described by magnitude alone; a number multiplied by each element of a vector or matrix.

Scale (multiplication): To compare the size of a product to the size of one factor on the basis of the size of the other factor.

Example*:* Compare the area of these rectangles. When one dimension is doubled, the area (A) is doubled.



Scale factor: The common factor by which distances of points from a given center in a dilation are multiplied.

Scatter plot: A graph in the coordinate plane representing a set of bivariate data. Example: the heights and weights of a group of people could be displayed on a scatter plot, where the heights are the *x*-coordinates of the points and the weights for the *y*-coordinates.

Scientific notation: A way of writing very large or very small numbers using a number between 1 and 10 multiplied by a power of 10.

Shape of distribution (Statistics and Probability, Grades 9-12): A description of a distribution. Examples: number of peaks, symmetry, skewness, or uniformity.

Similar figures: Two plane or solid figures which can be obtained from each other by a similarity motion (a sequence of dilations, rotations, reflections, and/or translations). Congruent figures are similar, with a scale factor of 100%.

Similarity motion: A geometric transformation of points consisting of a sequence of one or more dilations, translations, reflections, and/or rotations. Similarity motions preserve angle measure and change lengths proportionally. The constant ratio comparing side lengths is called the **scale factor**.

Simulation: The process of using a mathematical model to recreate real phenomenon, often repeatedly, so that the likelihood of various outcomes can be more accurately estimated.

Simultaneous equations: See *system of equations*.

Skewed: Term which describes a distribution of data that is not symmetrical about its mean.

Slope (rate of change): The ratio of vertical change to horizontal change; the ratio of change in an independent variable compared to change in the dependent variable.

Solid figure: A three-dimensional object.

Solution: A value which makes an equation or inequality true.

Solution to a system of equations: A solution that is common to each of the equations in the system.

Square root: A square root of x is the number that, when multiplied by itself, gives the number x. Example: 5 is the square root of 25 because $5 \cdot 5 = 25$.

Square root function: A function in which the output of the function is found by taking the square root of the input value, preceded or followed by addition, subtraction, or multiplication of the value by one or more constants. See Appendix D, Table 4.

Square unit: The area of a square with side lengths of 1 unit, used as a unit of measure for area.

Standard algorithm: A generally accepted method used to perform a particular mathematical computation.

Standard deviation: A measurement of dispersion that measures how far each number in the data set is from the mean. It is the square root of the variance divided by the number of cases. For the data set $\{x_1, x_2, x_3, \ldots, x_n\}$ with mean $x\bar{x}$, SD = $\sqrt{\frac{1}{m}\sum^{m}(xx - \bar{xx})^2}$. If the data set is a sample of a population, SD = $\sqrt{\frac{1}{nn-1}\sum^{nn}(xx - \bar{xx})^2}$.

Statistic: A characteristic of a sample used to estimate the value of a population parameter.

Statistically significant: An observed event is considered statistically significant when, in the presence of randomness, the observed results are not likely due to chance alone.

Stem-and-leaf plot: A graphical display of quantitative data that contains two columns separated by a vertical line. Each value is split into a stem and a leaf, the stem being the first digit(s) and the leaf being the last digit. The stems are typically given in the left column, and the leaves are given in order to the right.

Step function: A discontinuous piecewise defined function in which each piece is a horizontal line segment or a point. Example: greatest integer function.

Strategy: A plan or approach to find an answer or solve a problem.

Subject (Data Analysis, Statistics and Probability, Grades 9-12): An individual to which treatments are applied within a statistical experiment.

Sum: The result of quantities added together.

Supplementary angles: Two angles whose measures have a sum of 180˚.

Survey: A study that obtains data from a subset of a population to draw conclusions.

Symmetry: A quality a figure has when the figure is mapped onto itself by a rigid motion in which the points of the figure are not all mapped to themselves. Figures that have symmetry are said to be symmetric, and the rigid motion is called a symmetry of the figure. Example: equilateral triangles have a rotational symmetry of 120˚ since each vertex of an equilateral triangle will be mapped to another vertex when rotated 120˚ about the center of the triangle. Thus, equilateral triangles are said to be rotationally symmetric.

System of equations: A set of two or more equations that has a common set of solutions. *Solving the system* means finding those common solutions.

Tape diagram: A visual model that uses rectangles of uniform size to illustrate and solve a variety of problems, including number relationship and ratio problems.

Term: A single number or variable, or numbers and variables multiplied together.

Terminating decimal: A decimal number which can be expressed in a finite number of digits; a decimal that ends.

Theorem: A geometric statement that can be proven to be true based on definitions, axioms, and previously proven theorems.

Theoretical probability: The number of ways that an event can occur divided by the total number of outcomes from the sample space. Contrast to *experimental probability.*

Transitivity principle for indirect measurement: A measurement principle stating that if the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

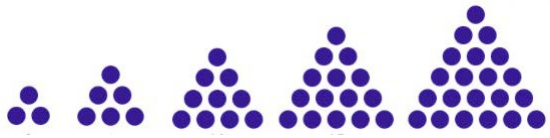
Transversal: A line that intersects two or more lines in a plane.

Treatment: A condition or set of conditions that is applied to one group in an experiment.

Treatment group (Statistics and Probability, Grades 9-12): The group of subjects to which the same treatment is assigned in an experiment.

Tree diagram: In probability, a diagram that shows all possible outcomes of an event.

Triangular numbers: The set of numbers *(1, 3, 6, 10, 15, …)* that are obtained from the summation of natural numbers that can be arranged in an equilateral triangle.



Trigonometric function: A function that relates the angles and sides of a right triangle, whose input is an angle and whose output is a designated ratio of sides. Trigonometric functions include sine, cosine, tangent, secant, cosecant, and cotangent.

Truth table: A table used to display all possible truth values of logical expressions.

Two-way table: A table listing the frequencies of two categorical variables whose values have been paired.

Uniform probability model: A probability model which assigns equal probability to all outcomes. See also *probability model.*

Unit circle: A circle with the equation $xx^2 + yy^2 = 1$. It has a radius of 1 unit.

Unit rate: In a proportional relationship, the number of units of the first quantity that correspond to one unit of the second quantity. Example: My rate of travel was 30 miles in one hour (mph).

Univariate data: A data set that is described by one variable (one type of data).

Variable: A symbol used to represent a quantity, which may have a fixed value or have changing values. In statistics, a variable is a characteristic of members of a population that can be measured (quantitative) or counted (categorical). Examples: The height of a person is a quantitative variable, while eye color is a categorical variable.

Variability: A measure of the spread of a data set. Examples: interquartile range, mean absolute deviation, standard deviation.

Variance: A measure of dispersion expressed in square units that considers how far each number in a data set is from its mean, calculated by finding the sum of the square of the distance of each value from the mean. Example: For the data set $\{x_1, x_2, x_3, \ldots, x_n\}$ with mean $x\bar{x}$, the variance = $\sum_{ii=1}^{nn} (xx_{ii} - x\bar{x})^2$.

Vector: A quantity with magnitude and direction in the plane or in space. It can be defined by an ordered pair or triple of real numbers.

Vertex: The common endpoint of two or more rays or line segments. Plural: vertices

Vertex form: The equation describing a quadratic equation using the vertex (maximum or minimum point) of its graph, which is a parabola. If (h, k) is the vertex of the graph represented by an equation, then the vertex form for that equation will be $f(x) = a(x - h)^2 + k$, where a is a non-zero constant.

Vertical angles: Two nonadjacent angles formed by a pair of intersecting lines.

Visual fraction model: A method of showing fractions. Examples: tape diagram, number line diagram, area model.

Volume: The measure of the amount of space inside of a solid figure, such as a cube, ball, cylinder, or pyramid, expressed in cubic units.

Whole numbers: The numbers 0, 1, 2, 3, …. without any decimal or fractional parts.

***x*-intercept:** The x-coordinate of the ordered pair where a graph crosses the x-axis, where $y = 0$.

***y*-intercept:** The y-coordinate of the ordered pair where a graph crosses the y-axis, where $x = 0$.

Zero matrix: A matrix consisting of all zeros. Any matrix added to a zero matrix of matching dimension will yield itself.

Zero of a function: A value of the independent variable where the value of the function equals 0.

Zero property: The product of any number and zero is zero.